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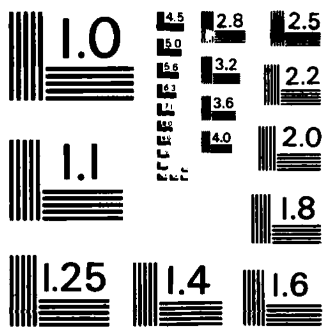

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The reduction of the lowest order nonlinear bremsstrahlung recoil force on the bare charge of an unperturbed relativistic test particle in a nonequilibrium beam-plasma system is derived from first principles. The purpose is to identify many of the important physical assumptions and mathematical techniques involved in Tsytovich's theory of nonlinear bremsstrahlung and radiative unstability.

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CONTENTS

1. INTRODUCTION

For a nonequilibrium beam-plasma system, the lowest-order Born approximation for the nonlinear force $\vec{F}_\alpha^{(1)}$ on the bare charge of an unperturbed relativistic test particle is given by[1-3]

$$\vec{F}_\alpha^{(1)} = \lim_{t\to\infty} \frac{2\pi}{t} e_\alpha \int dk \; \vec{k} \, \frac{\vec{v}_\alpha \cdot \vec{E}_k}{\omega + i\delta} \, \delta\left(\omega - \vec{k}\cdot\vec{v}_\alpha\right) \quad . \tag{1}$$

Here α is a label designating the species of the test particle of charge e_α and velocity $\vec{v}_\alpha$, t is the time, $\vec{k}$ is the wave vector, and ω is the frequency corresponding to the Fourier component $\vec{E}_k$ of the total field, $\delta(x)$ is the one-dimensional Dirac delta function, δ is a small imaginary part, and $dk \equiv d^3\vec{k}\, d\omega$. Equation (1) differs from equation (7) of Akopyan and Tsytovich[1] by an additional factor of $(2\pi)^{-3}$ appearing in the latter because of the different Fourier transform convention chosen there.[2] If the part of the contribution to the force, equation (1), which is associated with recoil of the unperturbed bare test particle due to bremsstrahlung, is averaged over the random phase of the bremsstrahlung field, it reduces to

$$\langle \vec{F}_\alpha^{\sigma\mathrm{rad}(1)} \rangle = \langle \vec{F}_\alpha^{\sigma\mathrm{rad}(1a)} \rangle + \langle \vec{F}_\alpha^{\sigma\mathrm{rad}(1b)} \rangle \quad , \tag{2}$$

where

$$\begin{aligned}\langle \vec{F}_\alpha^{\sigma\mathrm{rad}(1a)} \rangle = {} & \frac{1}{16\pi^2} e_\alpha^2 e_\beta^2 \int \frac{d^3\vec{p}_\beta \, d\omega \, d^3\vec{k} \, d^3\vec{\kappa}}{(2\pi)^3} \frac{\vec{\kappa}}{\omega^2 \left(\vec{\kappa}\cdot\vec{v}_\alpha\right)^4} \\ & \times \left|E_k^{\sigma(0)}\right|^2 \left| e_{kj}^{\sigma} \Lambda_{ij}^{(\beta)}(k,-\kappa) G_{ip}(-\kappa) v_{\alpha p}\right|^2 \\ & \times \left(\vec{\kappa} - \vec{k}\right)\cdot\vec{\nabla}_{p_\beta} f_{p_\beta}^{R(0)} \, \delta\left(\omega - \vec{\kappa}\cdot\vec{v}_\alpha + \left(\vec{\kappa} - \vec{k}\right)\cdot\vec{v}_\beta\right)\end{aligned} \tag{3}$$

is defined and calculated in this report, and $\langle \vec{F}_\alpha^{\sigma\mathrm{rad}(1b)} \rangle$ is defined by equations (25), (26), and (47) below but calculated in another report.* The quantity $\langle \vec{F}_\alpha^{\sigma\mathrm{rad}(1b)} \rangle$ is not mentioned by Akopyan and Tsytovich,[1] but it is nonvanishing and makes an important contribution to nonlinear bremsstrahlung.

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[1] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys. 1 (1975), 371].*

[2] *H. E. Brandt, Nonlinear Force on an Unpolarized Relativistic Test Particle to Second-Order in the Total Field in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1995 (August 1983).*

[3] *H. E. Brandt, Nonlinear Dynamic Polarization Force on a Relativistic Test Particle in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1994 (September 1983).*

**H. E. Brandt, Collective Bremsstrahlung Recoil Force on the Bare Charge of an Unperturbed Test Particle in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, 1983, (to be published).*

The quantity $\langle \vec{F}_\alpha^{\,\mathrm{rad}(1a)} \rangle$ given by equation (3) corresponds to equation (27) of Akopyan and Tsytovich.[1] Their equation (27) is expressed in Gaussian units, whereas equation (3) above is in MKS units. This results in an additional overall factor of $(4\pi)^2$ in Akopyan and Tsytovich.[1] Also there is an additional factor of $(2\pi)^{-6}$ in their work because of different Fourier transform conventions. However, their equation (27) also differs from equation (3) here in that they omit an overall factor of 2. Also, the arguments k and $-\kappa$ of $\Lambda_{ij}^{(\beta)}(k,-\kappa)$ in equation (3) appear incorrectly interchanged in equation (27) of Akopyan and Tsytovich.[1] As shown below, these disparities are evidently due to minor errors in their work.

In equation (3), β designates the species of the other particle with velocity $\vec{v}_\beta$, momentum $\vec{p}_\beta$, and background distribution $f_{p_\beta}^{R(0)}$ involved in the bremsstrahlung process. Also, $\vec{\kappa}$ is the momentum transfer from the test particle and $E_k^{\sigma(0)}$ is the amplitude of the lowest order stochastic bremsstrahlung field with polarization $\vec{e}_k^{\,\sigma}$ and mode σ. The matrix $\Lambda_{ij}^{(\beta)}(k,-\kappa)$ in equation (3) is defined by[1-3]

$$\Lambda_{ij}^{(\alpha)}(k_1,k) = \frac{e_\alpha}{\gamma_\alpha m_\alpha}\left(\delta_{ij} + \frac{v_{\alpha i}k_j - v_{\alpha j}k_{1i}}{\omega - \vec{k}\cdot\vec{v}_\alpha - i\delta} - \frac{v_{\alpha i}v_{\alpha j}\left(\vec{k}\cdot\vec{k}_1 - \frac{\omega\omega_1}{c^2}\right)}{\left(\omega - \vec{k}\cdot\vec{v}_\alpha - i\delta\right)^2}\right) . \tag{4}$$

The quantity $G_{mn}(k)$ in equation (3) is the linear photon Green's function, whose inverse is given by[4]

$$G_{ij}^{-1} = \frac{1}{\mu_o(\omega + i\delta)^2}\left(k_i k_j - k^2\delta_{ij}\right) + \varepsilon_{ij} . \tag{5}$$

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[1]*A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys. 1 (1975), 371].*

[2]*H. E. Brandt, Nonlinear Force on an Unpolarized Relativistic Test Particle to Second-Order in the Total Field in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1995 (August 1983).*

[3]*H. E. Brandt, Nonlinear Dynamic Polarization Force on a Relativistic Test Particle in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1994 (September 1983).*

[4]*H. E. Brandt, The Total Field in Collective Bremsstrahlung in a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1996 (September 1983).*

For a spatially isotropic system, the dielectric permittivity tensor ε_{ij} is given by[4-7]

$$\varepsilon_{ij}(\vec{k},\omega) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\varepsilon_t(\vec{k},\omega) + \frac{k_i k_j}{k^2}\,\varepsilon_\ell(\vec{k},\omega) \quad , \tag{6}$$

where ε_t and ε_ℓ are the transverse and longitudinal permittivity, respectively. In this case the linear photon Green's function is given by[4-6]

$$G_{ij} = \frac{k_i k_j}{k^2 \varepsilon_\ell} + \frac{\omega^2}{\varepsilon_t(\omega + i\delta)^2 - \frac{k^2}{\mu_o}}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \quad , \tag{7}$$

and the transverse and longitudinal permittivities are given by

$$\varepsilon_t = \varepsilon_o + \frac{1}{2}\,\frac{1}{(\omega + i\delta)^2}\sum_s \frac{e_s^2}{k^2}\int \frac{d^3\vec{p}_s}{(2\pi)^3}\left[k^2 v_{sm} + \frac{k^2 v_s^2 - \omega\vec{k}\cdot\vec{v}_s}{\omega - \vec{k}\cdot\vec{v}_s + i\delta}\,k_m\right]\frac{\partial f_{p_s}^{R(0)}}{\partial p_{sm}} \tag{8}$$

and

$$\varepsilon_\ell = \varepsilon_o + \frac{1}{\omega + i\delta}\sum_s \frac{e_s^2}{k^2}\int \frac{d^3\vec{p}_s}{(2\pi)^3}\,\frac{1}{\omega - \vec{k}\cdot\vec{v}_s + i\delta}\,\vec{k}\cdot\vec{v}_s\;\vec{k}\cdot\vec{\nabla}_{p_s} f_{p_s}^{R(0)} \quad , \tag{9}$$

respectively. Here ε_o and μ_o are the permittivity and permeability, respectively, of the vacuum. Each species s of particles present in the system makes a contribution to equations (8) and (9). If the distribution function depends only on energy, then using the expression ε_s for the relativistic energy of a particle of species s, namely,

$$\varepsilon_s = \left(p_s^2 c^2 + m_s^2 c^4\right)^{1/2} \quad , \tag{10}$$

allows equations (8) and (9) to be easily rewritten in terms of $\partial f_{p_s}^{R(0)}/\partial\varepsilon_s$.[4,5]

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[4]*H. E. Brandt, The Total Field in Collective Bremsstrahlung in a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1996 (September 1983).*

[5]*A. I. Akhiezer, I. A. Akhiezer, R. V. Polovin, A. G. Sitenko, and K. N. Stepanov,* Plasma Electrodynamics, *Vol. 1, Linear Theory, Pergamon Press (1975), 206.*

[6]*V. N. Tsytovich, Nonlinear Effects in Plasma, Plenum Press, New York (1970), 314.*

[7]*V. N. Tsytovich, Theory of Turbulent Plasma (Consultants Bureau), Plenum Publishing Corp., New York (1977), 63-65.*

In the present work, equations (2) and (3) are derived from first principles for the purpose of identifying some of the important physical assumptions and mathematical techniques involved in Tsytovich's theory of nonlinear bremsstrahlung and radiative instability in relativistic nonequilibrium beam-plasma systems (see also Selected Bibliography).[1] This will facilitate understanding and illustrate the procedures for reducing other components of the total nonlinear recoil force on a test particle due to bremsstrahlung emission.[1]

The present work, together with other related work by the author (see also Selected Bibliography),[2-4] is important for ongoing work in calculating collective radiative processes and conditions for the occurrence of radiative instability in relativistic nonequilibrium beam-plasma systems.

2. REDUCTION OF A COMPONENT OF THE BREMSSTRAHLUNG RECOIL FORCE

In this section, equation (3) is to be derived by reducing that part of equation (1) which corresponds to bremsstrahlung recoil. Also the quantity $\langle \vec{F}_{\alpha}^{\sigma \mathrm{rad}(1b)} \rangle$ appearing in equation (2) will be defined. It has been shown by using the balance equations that the general form for the collective bremsstrahlung recoil force on a test particle participating in induced bremsstrahlung in a nonequilibrium beam-plasma system is given by[1,8]

$$\vec{F}_{\alpha}^{\sigma} = -4\pi^3 \int \frac{d^3\vec{k}\, d\omega\, d^3\vec{\kappa}\, d^3\vec{p}_{\beta}}{(2\pi)^9} \frac{1}{\omega^2} \frac{\partial}{\partial \omega} \left(\omega^2 \varepsilon^{\sigma}(\vec{k},\omega)\right)\vec{\kappa}\left(\vec{\kappa} - \vec{k}\right)\cdot\left(\vec{\nabla}_{p_\beta} f_{p_\beta}\right) \qquad (11)$$

$$\times \left|E_{\vec{k}}^{\sigma(0)}\right|^2 v^{\sigma}_{p_\alpha, p_\beta}(\vec{\kappa},\vec{k})\,\delta\left(\omega - \vec{\kappa}\cdot\vec{v}_{\alpha} + \left(\vec{\kappa} - \vec{k}\right)\cdot\vec{v}_{\beta}\right) \quad .$$

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[1] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys. 1 (1975), 371].*

[2] *H. E. Brandt, Nonlinear Force on an Unpolarized Relativistic Test Particle to Second-Order in the Total Field in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1995 (August 1983).*

[3] *H. E. Brandt, Nonlinear Dynamic Polarization Force on a Relativistic Test Particle in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1994 (September 1983).*

[4] *H. E. Brandt, The Total Field in Collective Bremsstrahlung in a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1996 (September 1983).*

[8] *H. E. Brandt, On the Form of the Collective Bremsstrahlung Recoil Force in a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories HDL-TR-2026 (January 1984).*

Here the dielectric constant $\varepsilon^{\sigma}(\vec{k},\omega)$ for the bremsstrahlung mode σ is defined in terms of the dielectric permittivity tensor $\varepsilon_{ij}(k,\omega)$ and the unit electric polarization vector $\vec{e}_k^{\sigma}$ by[4,6,8,*]

$$\varepsilon^{\sigma}(\vec{k},\omega) = e_{ki}^{\sigma *}\varepsilon_{ij}e_{kj}^{\sigma} + \varepsilon_o \frac{c^2}{\omega^2}\left(\vec{k}\cdot\vec{e}_k^{\sigma}\right)\left(\vec{k}\cdot\vec{e}_k^{\sigma *}\right) \quad . \tag{12}$$

Also in equation (11), $E_k^{\sigma(0)}$ is the zeroth-order amplitude of the stochastic bremsstrahlung field,[4] and $V_{p_\alpha,p_\beta}^{\sigma}(\vec{k},\vec{k})$ is the bremsstrahlung transition probability with the delta function factored out, expressing conservation of energy. It is clear that $V_{p_\alpha,p_\beta}^{\sigma}(\vec{k},\vec{k})$ must be at least first order in the regular part of the field. In reducing equation (1), only those contributions which are of the same form as equation (11) are to be included, since only they correspond to bremsstrahlung.

The total field $\vec{E}_k$ appearing in equation (1) and associated with the bremsstrahlung process involving the test particle is given by[1,4]

$$\vec{E}_k = \vec{E}_k^{\sigma(0)} + \vec{E}_k^{(1)} + \vec{E}_k^{(2)} + \vec{E}_{dpk}^{(1)} + \vec{E}_{dpk}^{(2)} \quad . \tag{13}$$

Here $\vec{E}_k^{\sigma(0)}$ is the zeroth-order bremsstrahlung field

$$\vec{E}_k^{\sigma(0)} = E_k^{\sigma(0)}\vec{e}_k^{\sigma} \quad , \tag{14}$$

where σ designates the mode and $\vec{e}_k^{\sigma}$ is the polarization vector. If the bremsstrahlung field is to be real, then

$$\vec{E}_k^{\sigma(0)} = \vec{E}_{-k}^{\sigma(0)*} \quad . \tag{15}$$

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[1]*A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys. 1 (1975), 371].*

[4]*H. E. Brandt, The Total Field in Collective Bremsstrahlung in a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1996 (September 1983).*

[6]*V. N. Tsytovich, Nonlinear Effects in Plasma, Plenum Press, New York (1970), 314.*

[8]*H. E. Brandt, On the Form of the Collective Bremsstrahlung Recoil Force in a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-2026 (January 1984)*

**H. E. Brandt, Bremsstrahlung Recoil Force on the Third-Order Nonlinear Dynamic Polarization Charge of a Relativistic Test Particle, preprint, Harry Diamond Laboratories, 1983 (to be published).*

The fields $\vec{E}_k^{(1)}$ and $\vec{E}_k^{(2)}$ in equation (13) are the self fields arising directly from the relativistic test particle's own motion, namely,[4]

$$E_{kn}^{(1)} = \frac{-ie_\alpha}{(2\pi)^3(\omega + i\delta)} v_{\alpha m} G_{nm}(k)\,\delta(\omega - \vec{k}\cdot\vec{v}_\alpha) \tag{16}$$

and

$$E_{kn}^{(2)} = \frac{-e_\alpha}{(2\pi)^3(\omega + i\delta)} G_{nm}(k) \times \int \frac{dk_1}{\omega_1 - i\delta} E^*_{k_1 j} \Lambda_{mj}^{(\alpha)*}(k_1,k)\,\delta(\omega + \omega_1 - \vec{k}\cdot\vec{v}_\alpha - \vec{k}_1\cdot\vec{v}_\alpha) \quad . \tag{17}$$

The fields $\vec{E}_{dpk}^{(1)}$ and $\vec{E}_{dpk}^{(2)}$ in equation (13) are the increasing order fields produced by the dynamic polarization current induced by the test particle and are given by[4]

$$E_{dpkn}^{(1)} = \frac{i}{2(\omega + i\delta)} G_{nm}(k) \sum_s e_s \int \frac{dk_1\, dk_2}{(\omega_1 - i\delta)(\omega_2 - i\delta)} \delta(k + k_1 + k_2) \times \left[S_{mj\ell}^{(s)}(k,-k_1,-k_2) + S_{m\ell j}^{(s)}(k,-k_2,-k_1)\right] E^*_{k_1 j} E^*_{k_2 \ell} \tag{18}$$

and

$$E_{dpkn}^{(2)} = \frac{-i}{\omega + i\delta} G_{nm}(k) \sum_s e_s \int \frac{dk_1\, dk_2\, dk_3}{(\omega_1 - i\delta)(\omega_2 - i\delta)(\omega_3 - i\delta)} \times \delta(k + k_1 + k_2 + k_3)\, \Sigma_{mij\ell}^{(s)}(k,-k_1,-k_2,-k_3) E^*_{k_1 i} E^*_{k_2 j} E^*_{k_3 \ell} \quad , \tag{19}$$

respectively. The symmetrization in equation (19) appearing in earlier work[1,4] has been removed. The delta functions appearing in equations (18) and (19) are four-dimensional, namely,

$$\delta(k) \equiv \delta^3(\vec{k})\delta(\omega) \quad . \tag{20}$$

The second-order nonlinear conductivity tensor $S_{ij\ell}^{(s)}(k,k_1,k_2)$ appearing in equation (18) is given by

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[1]*A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys. 1 (1975), 371].*

[4]*H. E. Brandt, The Total Field in Collective Bremsstrahlung in a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1996 (September 1983).*

$$S^{(s)}_{ij\ell}(k,k_1,k_2) = e_s^2 \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{v_{si}}{\omega - \vec{k}\cdot\vec{v}_s + i\delta} \left[(\omega_1 - \vec{k}_1\cdot\vec{v}_s) \frac{\partial}{\partial p_{sj}} \right.$$

$$\left. + v_j k_{1m} \frac{\partial}{\partial p_{sm}} \right] \left[\frac{\partial}{\partial p_{s\ell}} + \frac{v_{s\ell}}{\omega_2 - \vec{k}_2\cdot\vec{v}_s + i\delta} k_{2n} \frac{\partial}{\partial p_{sn}} \right] f^{R(0)}_{ps} \quad . \tag{21}$$

The third-order nonlinear conductivity tensor $\Sigma^{(s)}_{ij\ell m}(k,k_1,k_2,k_3)$ is given by

$$\Sigma^{(s)}_{ij\ell m}(k,k_1,k_2,k_3)$$

$$= -ie_s^3 \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{\vec{v}_{si}}{\omega - \vec{k}\cdot\vec{v}_s + i\delta}$$

$$\times \left[\delta_{jn}(\omega_1 - \vec{k}_1\cdot\vec{v}_s) + k_{1n}v_{sj}\right] \frac{\partial}{\partial p_{sn}} \frac{1}{\omega - \omega_1 - (\vec{k} - \vec{k}_1)\cdot\vec{v}_s + i\delta} \tag{22}$$

$$\times \left[\delta_{\ell u}(\omega_2 - \vec{k}_2\cdot\vec{v}_s) + k_{2u}v_{s\ell}\right] \frac{\partial}{\partial p_{su}} \frac{1}{\omega_3 - \vec{k}_3\cdot\vec{v}_s + i\delta}$$

$$\times \left[\delta_{mq}(\omega_3 - \vec{k}_3\cdot\vec{v}_s) + k_{3q}v_{sm}\right] \frac{\partial}{\partial p_{sq}} f^{R(0)}_{p_s} \quad .$$

Letting $B\vec{F}^{(1)}_\alpha$ denote the operation of including only that part of $\vec{F}^{(1)}_\alpha$ which is of the bremsstrahlung form given by equation (11), we obtain

$$\langle \vec{F}^{\sigma rad(1)}_\alpha \rangle = B\langle \vec{F}^{(1)}_\alpha \rangle \quad . \tag{23}$$

Here < > denotes the ensemble average over the statistical phase distribution of the bremsstrahlung, which to the needed order is assumed to be random.[4] Substituting equations (1) and (13) in equation (23) gives

$$\langle \vec{F}^{\sigma rad(1)}_\alpha \rangle = \sum_{n=1}^{5} \langle \vec{F}^{\sigma rad(1)}_\alpha \rangle_n \quad , \tag{24}$$

where

$$\langle \vec{F}^{\sigma rad(1)}_\alpha \rangle_n = B \lim_{t\to\infty} \frac{2\pi}{t} e_\alpha \int \frac{dk}{\omega + i\delta} \vec{k}\ \vec{v}_\alpha \cdot \langle \vec{E}^n_k \rangle \delta(\omega - \vec{k}\cdot\vec{v}_\alpha) \tag{25}$$

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[4] *H. E. Brandt, The Total Field in Collective Bremsstrahlung in a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1996 (September 1983).*

and

$$\left\{\vec{E}_k^n,\ n = 1,\ 2,\ \ldots 5\right\} = \left\{\vec{E}_k^{\sigma(0)},\ \vec{E}_k^{(1)},\ \vec{E}_k^{(2)},\ \vec{E}_{dpk}^{(1)},\ \vec{E}_{dpk}^{(2)}\right\} \quad . \tag{26}$$

To the needed order, the stochastic properties of the bremsstrahlung field are approximated by[1,4]

$$\langle E_{ki}^{\sigma(0)} \rangle = 0 \tag{27}$$

and

$$\langle E_{ki}^{\sigma(0)} E_{k_1 j}^{\sigma(0)} \rangle = e_{ki}^{\sigma} e_{kj}^{\sigma *} \left| E_k^{\sigma(0)} \right|^2 \delta\left(k + k_1\right) \quad . \tag{28}$$

Substituting equations (26) and (27) in equation (25) for n = 1 produces

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_1 = 0 \quad . \tag{29}$$

If we substitute equations (26) and (16) in equation (25) for n = 2, it is evident that the resulting expression is not of the required form given by equation (11). For example it does not contain $\left|E_k^{\sigma(0)}\right|^2$; therefore the B operation gives zero, thus:

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_2 = 0 \quad . \tag{30}$$

Next substituting equations (26) and (17) in equation (25) for n = 3, we obtain

$$\begin{aligned} \langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_3 = B \lim_{t\to\infty} \frac{2\pi}{t} e_\alpha \int \frac{dk}{\omega + i\delta} \vec{k} v_{\alpha n} \delta\left(\omega - \vec{k}\cdot\vec{v}_\alpha\right) \left[\frac{-e_\alpha G_{nm}(k)}{(2\pi)^3(\omega + i\delta)}\right] \\ \times \int \frac{dk_1}{\omega_1 - i\delta} \langle E_{k_1 j}^{*} \rangle \Lambda_{mj}^{(\alpha)*}\left(k_1, k\right) \delta\left(\omega + \omega_1 - \vec{k}\cdot\vec{v}_\alpha - \vec{k}_1\cdot\vec{v}_\alpha\right) \quad . \end{aligned} \tag{31}$$

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[1] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys. 1 (1975), 371].*

[4] *H. E. Brandt, The Total Field in Collective Bremsstrahlung in a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1996 (September 1983).*

Furthermore, substituting equation (13) in equation (31) and using equation (26) then

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_3 = \sum_{n=1}^{5} \langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{3n} \quad , \tag{32}$$

where

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{3n} = B \lim_{t\to\infty} \frac{2\pi}{t} e_\alpha \int \frac{dk}{\omega + i\delta} \vec{k} v_{\alpha n} \delta\left(\omega - \vec{k}\cdot\vec{v}_\alpha\right) \left[\frac{-e_\alpha G_{nm}(k)}{(2\pi)^3(\omega + i\delta)}\right]$$
$$\times \int \frac{dk_1}{\omega_1 - i\delta} \langle E_{k_1 j}^{n*} \rangle \Lambda_{mj}^{(\alpha)*}(k_1, k)\delta\left(\omega + \omega_1 - \vec{k}\cdot\vec{v}_\alpha - \vec{k}_1\cdot\vec{v}_\alpha\right) \quad . \tag{33}$$

Substituting equations (26) and (27) in equation (33) for n = 1 produces

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{31} = 0 \quad . \tag{34}$$

Next substituting equations (26) and (16) in equation (33) for n = 2, we obtain an expression which is clearly not of the bremsstrahlung form given by equation (11), and therefore the B operation yields zero, thus:

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{32} = 0 \quad . \tag{35}$$

Next substituting equations (26) and (17) in equation (33) for n = 3, we obtain a higher order Born term which is also not of the bremsstrahlung form given by equation (11), and therefore

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{33} = 0 \quad . \tag{36}$$

For consistency with the Born approximation, only those terms which are first or second order in the regular part of the field are needed.[1]

So that in the limit of $t \to \infty$ the integrals in equation (25) or equation (33) do not vanish, it is important to note here that the following combination of delta functions is needed in the integrand:

$$\delta^2\left(\omega - \vec{k}\cdot\vec{v}_\alpha\right) = \lim_{t\to\infty} \frac{t}{2\pi} \delta\left(\omega - \vec{k}\cdot\vec{v}_\alpha\right) \quad . \tag{37}$$

In this way the t^{-1} factor is canceled. Equation (37) is obtained as follows. The well-known integral representation of the delta function is

$$\delta(\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{-i\omega t} \quad . \tag{38}$$

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[1] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys. 1 (1975), 371].*

Therefore

$$\delta(\omega - \vec{k}\cdot\vec{v}) = \lim_{t\to\infty} \int_{-t/2}^{t/2} \frac{dt'}{2\pi} e^{-i(\omega-\vec{k}\cdot\vec{v})t'} \quad . \tag{39}$$

Then using equation (39) one has

$$\delta^2(\omega - \vec{k}\cdot\vec{v}_\alpha) = \delta(\omega - \vec{k}\cdot\vec{v}_\alpha) \lim_{t\to\infty} \int_{-t/2}^{t/2} \frac{dt'}{2\pi} e^{-i(\omega-\vec{k}\cdot\vec{v}_\alpha)t'} \tag{40}$$

$$= \lim_{t\to\infty} \int_{-t/2}^{t/2} \frac{dt'}{2\pi} \delta(\omega - \vec{k}\cdot\vec{v}_\alpha) e^{-i(\omega-\vec{k}\cdot\vec{v}_\alpha)t'}$$

$$= \lim_{t\to\infty} \int_{-t/2}^{t/2} \frac{dt'}{2\pi} \delta(\omega - \vec{k}\cdot\vec{v}_\alpha)$$

$$= \lim_{t\to\infty} \frac{t}{2\pi} \delta(\omega - \vec{k}\cdot\vec{v}_\alpha) \quad .$$

Thus in evaluating equation (31) one has an integrand containing the factor

$$F = \frac{1}{t} \delta(\omega - \vec{k}\cdot\vec{v}_\alpha) \langle E^*_{k_1 j} \rangle \delta(\omega + \omega_1 - \vec{k}\cdot\vec{v}_\alpha - \vec{k}_1\cdot\vec{v}_\alpha) \tag{41}$$

or

$$F = \frac{1}{t} \delta(\omega - \vec{k}\cdot\vec{v}_\alpha) \langle E^*_{k_1 j} \rangle \delta(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha) \quad ; \tag{42}$$

or substituting equation (13) in equation (42) produces

$$F = \frac{1}{t} \sum_{n=1}^{5} \delta(\omega - \vec{k}\cdot\vec{v}_\alpha) \langle E^{n*}_{k_1 j} \rangle \delta(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha) \quad . \tag{43}$$

Because of equation (37), another factor of $\delta(\omega_1 - \vec{k}\cdot\vec{v}_\alpha)$ is needed to give a nonvanishing limit. For n = 2 in equation (43), one picks up a factor of $\delta(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha)$ by equation (16); however, the remaining integrand does not contain the factor $|E_k^{\sigma(0)}|^2$ necessary to be of the bremsstrahlung form given by equation (11). An iteration of equation (17) in equation (43) can give the quadratic delta function, but the remaining integrand is higher order. By analogous reasoning

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{34} = 0 \quad , \tag{44}$$

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{35} = 0 \quad . \tag{45}$$

Therefore, substituting equations (34) to (36), (44), and (45) in equation (32) produces

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_3 = 0 \quad . \tag{46}$$

The quantity $\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_4$ is nonvanishing and is calculated in a separate report.* Here the result of that calculation will be designated $\langle \vec{F}_\alpha^{\sigma rad(1b)} \rangle$. Thus

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_4 \equiv \langle \vec{F}_\alpha^{\sigma rad(1b)} \rangle \quad . \tag{47}$$

In the present report we emphasize the calculation of $\langle \vec{F}^{\sigma rad(1a)} \rangle$ in equation (2).

Substituting equations (26) and (19) in equation (25) one has

$$\begin{aligned} \langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_5 = B \lim_{t\to\infty} \frac{2\pi}{t} e_\alpha \int \frac{dk}{\omega + i\delta} \vec{k}\delta\left(\omega - \vec{k}\cdot\vec{v}_\alpha\right) v_{\alpha n} \\ \times \frac{-i}{\omega + i\delta} G_{nm}(k) \sum_s e_s \int \frac{dk_1 \; dk_2 \; dk_3}{\left(\omega_1 - i\delta\right)\left(\omega_2 - i\delta\right)\left(\omega_3 - i\delta\right)} \\ \times \delta\left(k + k_1 + k_2 + k_3\right) \Sigma_{mij\ell}^{(s)}(k,-k_1,-k_2,-k_3) \langle E_{k_1 i}^* E_{k_2 j}^* E_{k_3 \ell}^* \rangle \quad . \end{aligned} \tag{48}$$

One has, using equation (13) to the required order in equation (48),

$$\begin{aligned} \langle E_{k_1 i} E_{k_2 j} E_{k_3 \ell} \rangle = \langle \left(E_{k_1 i}^{\sigma(0)} + E_{k_1 i}^{(1)} + E_{k_1 i}^{(2)} + E_{dpki}^{(1)} + E_{dpki}^{(2)}\right) \\ \times \left(E_{k_2 j}^{\sigma(0)} + E_{k_2 j}^{(1)} + E_{k_2 j}^{(2)} + E_{dpk_2 j}^{(1)} + E_{dpk_2 j}^{(2)}\right) \\ \times \left(E_{k_3 \ell}^{\sigma(0)} + E_{k_3 \ell}^{(1)} + E_{k_3 \ell}^{(2)} + E_{dpk_3 \ell}^{(1)} + E_{dpk_3 \ell}^{(2)}\right) \rangle \quad . \end{aligned} \tag{49}$$

If equation (49) is substituted in equation (48), the only possible nonvanishing contributions to the required order are given by

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**H. E. Brandt, Collective Bremsstrahlung Recoil Force on the Bare Charge of an Unperturbed Test Particle in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, 1983 (to be published as HDL technical report).*

$$\langle \vec{F}_{\alpha}^{\sigma rad(1)} \rangle_5$$

$$= B \lim_{t\to\infty} \frac{2\pi}{t} e_{\alpha} \int \frac{dk}{\omega + i\delta} \vec{k}\ \delta(\omega - \vec{k}\cdot\vec{v}_{\alpha}) v_{\alpha p}$$

$$\times \left(\frac{-i}{\omega + i\delta}\right) G_{pm}(k) \sum_s e_s \int \frac{dk_1\ dk_2\ dk_3}{(\omega_1 - i\delta)(\omega_2 - i\delta)(\omega_3 - i\delta)}$$

$$\times \delta(k + k_1 + k_2 + k_3) \Sigma^{(s)}_{mij\ell}(k,-k_1,-k_2,-k_3) \left\{\frac{1}{2} E^{(1)*}_{k_3\ell} \langle E^{\sigma(0)*}_{k_1 i} E^{\sigma(0)*}_{k_2 j} \rangle\right.$$

$$\left. + \frac{1}{2} E^{(1)*}_{k_2 j} \langle E^{\sigma(0)*}_{k_1 i} E^{\sigma(0)*}_{k_3\ell} \rangle + \frac{1}{2} E^{(1)*}_{k_1 i} \langle E^{\sigma(0)*}_{k_2 j} E^{\sigma(0)*}_{k_3\ell} \rangle \right\} . \tag{50}$$

The factors of 1/2 in the three terms in the braces arise from the fact that equation (50) is effectively a self force; the field $E^{(1)}_{kn}$ in equation (50) and given by equation (16) is a field produced by the test particle α, and through the dynamic polarization that it induces it produces a force which acts back on the same test particle. Equation (50) may be rewritten as

$$\langle \vec{F}_{\alpha}^{\sigma rad(1)} \rangle_5 = \sum_{n=1}^{3} \langle \vec{F}_{\alpha}^{\sigma rad(1)} \rangle_{5n} , \tag{51}$$

where

$$\langle \vec{F}_{\alpha}^{\sigma rad(1)} \rangle_{51}$$

$$= B \lim_{t\to\infty} \frac{2\pi}{t} e_{\alpha} \int \frac{dk}{\omega + i\delta} \vec{k}\delta(\omega - \vec{k}\cdot\vec{v}_{\alpha}) v_{\alpha p}$$

$$\times \left(\frac{-i}{\omega + i\delta}\right) G_{pm}(k) \sum_s e_s \int \frac{dk_1\ dk_2\ dk_3}{(\omega_1 - i\delta)(\omega_2 - i\delta)(\omega_3 - i\delta)} \tag{52}$$

$$\times \delta(k + k_1 + k_2 + k_3) \Sigma^{(s)}_{mij\ell}(k,-k_1,-k_2,-k_3)\ \frac{1}{2} E^{(1)*}_{k_3\ell} \langle E^{\sigma(0)*}_{k_1 i} E^{\sigma(0)*}_{k_2 j} \rangle$$

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{52}$$

$$= B \lim_{t\to\infty} \frac{2\pi}{t} e_\alpha \int \frac{dk}{\omega + i\delta} \vec{k}\delta(\omega - \vec{k}\cdot\vec{v}_\alpha) v_{\alpha p}$$
$$\times \left(\frac{-i}{\omega + i\delta}\right) G_{pm}(k) \sum_s e_s \int \frac{dk_1\, dk_2\, dk_3}{(\omega_1 - i\delta)(\omega_2 - i\delta)(\omega_3 - i\delta)} \tag{53}$$
$$\times \delta(k + k_1 + k_2 + k_3)\Sigma^{(s)}_{mij\ell}(k,-k_1,-k_2,-k_3)\ \frac{1}{2}\ E^{(1)*}_{k_2 j}\ \langle E^{\sigma(0)*}_{k_1 i}\ E^{\sigma(0)*}_{k_3 \ell} \rangle$$

and

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{53}$$

$$= B \lim_{t\to\infty} \frac{2\pi}{t} e_\alpha \int \frac{dk}{\omega + i\delta} \vec{k}\delta(\omega - \vec{k}\cdot\vec{v}_\alpha) v_{\alpha p}$$
$$\times \left(\frac{-i}{\omega + i\delta}\right) G_{pm}(k) \sum_s e_s \int \frac{dk_1\, dk_2\, dk_3}{(\omega_1 - i\delta)(\omega_2 - i\delta)(\omega_3 - i\delta)} \tag{54}$$
$$\times \delta(k + k_1 + k_2 + k_3)\Sigma^{(s)}_{mij\ell}(k,-k_1,-k_2,-k_3)\ \frac{1}{2}\ E^{(1)*}_{k_1 i} \langle E^{\sigma(0)*}_{k_2 j} E^{\sigma(0)*}_{k_3 \ell} \rangle \quad .$$

Substituting equations (28) and (16) in equation (52) produces

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{51}$$

$$= B \lim_{t\to\infty} \frac{2\pi}{t} e_\alpha \int \frac{dk}{\omega + i\delta} \vec{k}\delta(\omega - \vec{k}\cdot\vec{v}_\alpha)$$
$$\times v_{\alpha p} \left(\frac{-i}{\omega + i\delta}\right) G_{pm}(k) \sum_s e_s \int \frac{dk_1\, dk_2\, dk_3}{(\omega_1 - i\delta)(\omega_2 - i\delta)(\omega_3 - i\delta)} \tag{55}$$
$$\times \delta(k + k_1 + k_2 + k_3)\Sigma^{(s)}_{mij\ell}(k,-k_1,-k_2,-k_3)$$
$$\times \frac{1}{2} \left[\frac{ie_\alpha}{(2\pi)^3(\omega_3 - i\delta)}\right] v_{\alpha n} G^*_{\ell n}(k_3)\delta(\omega_3 - \vec{k}_3\cdot\vec{v}_\alpha) e^{\sigma*}_{k_1 i} e^{\sigma}_{k_1 j} |E^{\sigma(0)}_{k_1}|^2$$
$$\times \delta(k_1 + k_2) \quad .$$

Integrating equation (55) first over k_2, using the property of the delta function $\delta(k_1 + k_2)$ and effectively setting $k_2 = -k_1$ (that is, $\vec{k}_2 = -\vec{k}_1$, $\omega_2 = -\omega_1$), we obtain

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{51} = B \lim_{t\to\infty} \frac{-e_\alpha^2}{8\pi^2 t} \int \frac{dk\ dk_1\ dk_3\ \vec{k}\delta(k + k_3)\delta(\omega - \vec{k}\cdot\vec{v}_\alpha)}{(\omega + i\delta)^2(\omega_1 - i\delta)(\omega_1 + i\delta)(\omega_3 - i\delta)^2}$$

$$\times\ v_{\alpha p} G_{pm}(k) \sum_s e_s \Sigma^{(s)}_{mij\ell}(k,-k_1,k_1,-k_3) \tag{56}$$

$$\times\ v_{\alpha n} G^*_{\ell n}(k_3)\delta(\omega_3 - \vec{k}_3\cdot\vec{v}_\alpha) e^{\sigma *}_{k_1 i} e^{\sigma}_{k_1 j} |E^{\sigma(0)}_{k_1}|^2 \ .$$

Next integrating equation (56) over k_3 produces

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{51} = B \lim_{t\to\infty} \frac{-e_\alpha^2}{8\pi^2 t} \int \frac{dk\ dk_1}{(\omega + i\delta)^4 \omega_1^2} \vec{k}\delta^2(\omega - \vec{k}\cdot\vec{v}_\alpha)$$

$$\times\ v_{\alpha p} G_{pm}(k) \sum_s e_s \Sigma^{(s)}_{mij\ell}(k,-k_1,k_1,k) v_{\alpha n} G^*_{\ell n}(-k) \tag{57}$$

$$\times\ e^{\sigma *}_{k_1 i} e^{\sigma}_{k_1 j} |E^{\sigma(0)}_{k_1}|^2 \ .$$

One also has

$$G^*_{nm}(-k) = G_{nm}(k) \ . \tag{58}$$

Equation (58) holds since[4]

$$E_{kn} = -\frac{i}{\omega + i\delta} G_{nm}(k) j_{km} \tag{59}$$

and by the reality of the current and field

$$E^*_{-kn} = E_{kn} \tag{60}$$

and

$$j^*_{-kn} = j_{kn} \ . \tag{61}$$

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[4] *H. E. Brandt, The Total Field in Collective Bremsstrahlung in a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1996 (September 1983).*

Taking the complex conjugate of equation (59), replacing k by -k, and substituting equations (60) and (61) produces

$$E_{kn} = \frac{-i}{\omega + i\delta} G^{*}_{nm}(-k) j_{km} \quad . \tag{62}$$

Comparing equations (59) and (62) shows that equation (58) follows. Next substituting equations (37) and (58) in equation (57), and noting that the t's cancel, we find that the limit is trivial and

$$\begin{aligned} \langle \vec{F}_{\alpha}^{\sigma rad(1)} \rangle_{51} = {} & B\left(\frac{-e_{\alpha}^{2}}{16\pi^{3}}\right) \int \frac{dk \; dk_1 \; \vec{k}\,\delta(\omega - \vec{k}\cdot\vec{v}_{\alpha})}{(\omega + i\delta)^{4}\omega_1^{2}} \\ & \times v_{\alpha p} G_{pm}(k) \sum_{s} e_s \Sigma^{(s)}_{mij\ell}(k,-k_1,k_1,k) v_{\alpha n} G_{\ell n}(k) \\ & \times e^{\sigma *}_{k_1 i} e^{\sigma}_{k_1 j} \left| E^{\sigma(0)}_{k_1} \right|^{2} \quad . \end{aligned} \tag{63}$$

Changing variables $\{k,k_1\}$ to $\{k_1,k\}$ and $\{i,j,\ell\}$ to $\{\ell,i,j\}$ in equation (63) produces

$$\begin{aligned} \langle \vec{F}_{\alpha}^{\sigma rad(1)} \rangle_{51} = {} & B\left(\frac{-e_{\alpha}^{2}}{16\pi^{3}}\right) \sum_{s} e_s \int \frac{dk \; dk_1}{\omega^{2}(\omega_1 + i\delta)^{4}} \vec{k}_1 \left| E^{\sigma(0)}_{k} \right|^{2} \\ & \times v_{\alpha p} G_{pm}(k_1) v_{\alpha n} G_{jn}(k_1) e^{\sigma *}_{k\ell} e^{\sigma}_{ki} \\ & \times \Sigma^{(s)}_{m\ell ij}(k_1,-k,k,k_1) \delta(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha}) \quad . \end{aligned} \tag{64}$$

If we substitute equations (28) and (16) in equation (53), we obtain

$$\begin{aligned} & \langle \vec{F}_{\alpha}^{\sigma rad(1)} \rangle_{52} \\ & = B \lim_{t\to\infty} \frac{2\pi}{t} e_{\alpha} \int \frac{dk}{\omega + i\delta} \vec{k}\,\delta(\omega - \vec{k}\cdot\vec{v}_{\alpha}) \\ & \times v_{\alpha p}\left(\frac{-i}{\omega + i\delta}\right) G_{pm}(k) \sum_{s} e_s \int \frac{dk_1 \; dk_2 \; dk_3}{(\omega_1 - i\delta)(\omega_2 - i\delta)(\omega_3 - i\delta)} \\ & \times \delta(k + k_1 + k_2 + k_3) \Sigma^{(s)}_{mij\ell}(k,-k_1,-k_2,-k_3) \, \frac{1}{2} \left[\frac{ie_{\alpha}}{(2\pi)^{3}(\omega_2 - i\delta)}\right] \\ & \times v_{\alpha n} G^{*}_{jn}(k_2) \delta(\omega_2 - \vec{k}_2\cdot\vec{v}_{\alpha}) e^{\sigma *}_{k_1 i} e^{\sigma}_{k_1 \ell} \left| E^{\sigma(0)}_{k_1} \right|^{2} \delta(k_1 + k_3) \quad . \end{aligned} \tag{65}$$

If we integrate over k_3 and k_2, equation (65) becomes

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{52} = B \lim_{t\to\infty} \left(\frac{-e_\alpha^2}{8\pi^2 t}\right) \int \frac{dk\ dk_1\ \vec{k}}{(\omega + i\delta)^4 \omega_1^2}$$

$$\times\ \delta^2(\omega - \vec{k}\cdot\vec{v}_\alpha) v_{\alpha p} G_{pm}(k) \sum_s e_s \Sigma_{mij\ell}^{(s)}(k,-k_1,k,k_1) \tag{66}$$

$$\times\ v_{\alpha n} G_{jn}^*(-k) e_{k_1 i}^{\sigma *} e_{k_1 \ell}^{\sigma} |E_{k_1}^{\sigma(0)}|^2 \quad .$$

Next substituting equations (37) and (58) in equation (66) produces

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{52} = B\left(\frac{-e_\alpha^2}{16\pi^3}\right) \int \frac{dk\ dk_1\ \vec{k}}{(\omega + i\delta)^4 \omega_1^2}\ \delta(\omega - \vec{k}\cdot\vec{v}_\alpha)$$

$$\times\ v_{\alpha p} G_{pm}(k) \sum_s e_s \Sigma_{mij\ell}^{(s)}(k,-k_1,k,k_1) v_{\alpha n} G_{jn}(k) \tag{67}$$

$$\times\ e_{k_1 i}^{\sigma *} e_{k_1 \ell}^{\sigma} |E_{k_1}^{\sigma(0)}|^2 \quad .$$

Changing variables $\{k_1,k\}$ to $\{k,k_1\}$ and $\{i,\ell\}$ to $\{\ell,i\}$ in equation (67), we obtain

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{52} = B\left(\frac{-e_\alpha^2}{16\pi^3}\right) \int \frac{dk\ dk_1\ \vec{k}_1}{\omega^2(\omega_1 + i\delta)^4}\ |E_k^{\sigma(0)}|^2$$

$$\times\ v_{\alpha p} G_{pm}(k_1) v_{\alpha n} G_{jn}(k_1) e_{k\ell}^{\sigma *} e_{ki}^{\sigma} \tag{68}$$

$$\times \sum_s e_s \Sigma_{m\ell ji}^{(s)}(k_1,-k,k_1,k)\delta(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha) \quad .$$

Next, substituting equations (28) and (16) in equation (54), we get

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{53} = B \lim_{t\to\infty} \frac{2\pi}{t} e_\alpha \int \frac{dk}{\omega + i\delta} \vec{k}\delta(\omega - \vec{k}\cdot\vec{v}_\alpha)$$

$$\times\ v_{\alpha p}\left(\frac{-i}{\omega + i\delta}\right) G_{pm}(k) \sum_s e_s \int \frac{dk_1\ dk_2\ dk_3}{(\omega_1 - i\delta)(\omega_2 - i\delta)(\omega_3 - i\delta)}$$

$$\times\ \delta(k + k_1 + k_2 + k_3)\Sigma_{mij\ell}^{(s)}(k,-k_1,-k_2,-k_3) \tag{69}$$

$$\times\ \frac{1}{2}\left[\frac{ie_\alpha}{(2\pi)^3(\omega_1 - i\delta)}\right] v_{\alpha n} G_{in}^*(k_1)\delta(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha)$$

$$\times\ e_{k_2 j}^{\sigma *} e_{k_2 \ell}^{\sigma} |E_{k_2}^{\sigma(0)}|^2 \delta(k_2 + k_3) \quad .$$

If we integrate over k_3 and k_1, equation (69) becomes

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{53} = B \lim_{t\to\infty} \left(\frac{-e_\alpha^2}{8\pi^2 t}\right) \int \frac{dk\, dk_2\, \vec{k}}{(\omega + i\delta)^4 \omega_2^2}$$

$$\times\, \delta^2(\omega - \vec{K}\cdot\vec{v}_\alpha) v_{\alpha p} G_{pm}(k) \sum_s e_s \Sigma^{(s)}_{mij\ell}(k,k,-k_2,k_2) \tag{70}$$

$$\times\, v_{\alpha n} G^*_{in}(-k) e^{\sigma *}_{k_2 j} e^{\sigma}_{k_2 \ell} |E^{\sigma(0)}_{k_2}|^2 \quad .$$

Next substituting equations (37) and (58) in equation (70) produces

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{53} = B \left(\frac{-e_\alpha^2}{16\pi^3}\right) \int \frac{dk\, dk_2\, \vec{k}}{(\omega + i\delta)^4 \omega_2^2}\, \delta(\omega - \vec{k}\cdot\vec{v}_\alpha)$$

$$\times\, v_{\alpha p} G_{pm}(k) \sum_s e_s \Sigma^{(s)}_{mij\ell}(k,k,-k_2,k_2) \tag{71}$$

$$\times\, v_{\alpha n} G_{in}(k) e^{\sigma *}_{k_2 j} e^{\sigma}_{k_2 \ell} |E^{\sigma(0)}_{k_2}|^2 \quad .$$

Changing variables $\{k,k_2\}$ to $\{k_1,k\}$ and $\{i,j,\ell\}$ to $\{j,\ell,i\}$ in equation (71) produces

$$\langle \vec{F}_\alpha^{\sigma rad(1)} \rangle_{53} = B \left(\frac{-e_\alpha^2}{16\pi^3}\right) \int \frac{dk\, dk_1\, \vec{k}_1}{\omega^2(\omega_1 + i\delta)^4} |E^{\sigma(0)}_{k}|^2$$

$$\times\, v_{\alpha p} G_{pm}(k_1) v_{\alpha n} G_{jn}(k_1) e^{\sigma *}_{k\ell} e^{\sigma}_{ki} \tag{72}$$

$$\times \sum_s e_s \Sigma^{(s)}_{mj\ell i}(k_1,k_1,-k,k)\, \delta(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha) \quad .$$

However, equation (72) is not of the bremsstrahlung form given by equation (11); in particular, it does not contain the delta function factor $\delta\left(\omega - \vec{k}\cdot\vec{v}_\alpha + (\vec{k} - \vec{K})\cdot\vec{v}_\beta\right)$ in the integrand. Such a factor could only appear through the third-order nonlinear conductivity tensor appearing in equation (72). According to equation (22), the factor $\left[\omega - \omega_1 - (\vec{K} - \vec{K}_1)\cdot\vec{v}_s + i\delta\right]^{-1}$ appears in $\Sigma^{(s)}_{ij\ell m}(k,k_1,k_2,k_3)$, and it can be written

$$\frac{1}{\omega - \omega_1 - (\vec{k} - \vec{k}_1)\cdot\vec{v}_s + i\delta} = P\, \frac{1}{\omega - \omega_1 - (\vec{k} - \vec{k}_1)\cdot\vec{v}_s} \tag{73}$$

$$- i\pi\delta\left(\omega - \omega_1 - (\vec{K} - \vec{K}_1)\cdot\vec{v}_s\right) \quad ,$$

and the imaginary part contains a delta function; however, in equation (72) it is $\Sigma^{(s)}_{mj\ell i}(k_1,k_1,-k,k)$ which appears, so that the delta function becomes $-i\pi\delta(0)$, which is not of the required form. Thus the B operation in equation (72) yields zero, and therefore

$$\langle \vec{F}^{\sigma rad(1)}_{\alpha} \rangle_{53} = 0 \quad . \tag{74}$$

Next substituting equations (64), (68), and (74) in equation (51) and the result together with equations (29), (30), (46), and (47) in equation (24), and comparing with equation (2), then

$$\begin{aligned} \langle \vec{F}^{\sigma rad(1a)}_{\alpha} \rangle = B \left(\frac{-e^2_{\alpha}}{16\pi^3}\right) \sum_s e_s \int \frac{dk\ dk_1\ \vec{k}_1}{\omega^2(\omega_1 + i\delta)^4} \left|E^{\sigma(0)}_{k}\right|^2 \\ \times v_{\alpha p} G_{pm}(k_1) v_{\alpha n} G_{jn}(k_1) e^{\sigma *}_{k\ell} e^{\sigma}_{ki} \delta(\omega_1 - \vec{k}_1 \cdot \vec{v}_{\alpha}) \\ \times \left[\Sigma^{(s)}_{m\ell ij}(k_1,-k,k,k_1) + \Sigma^{(s)}_{m\ell ji}(k_1,-k,k_1,k)\right] \quad . \end{aligned} \tag{75}$$

It remains to be shown in this report that equation (75) reduces to equation (3).

Evidently equation (75) should agree with equation (24) of Akopyan and Tsytovich.[1] Clearly there is a typographical error in the latter since a factor of e_{α} has been omitted. Also a factor of (-2) and complex conjugation signs on the conductivity tensor have apparently been omitted as shown below. Their equation (24)[1] contains an additional overall factor of $(4\pi)^2(2\pi)^{-6}$ due to the use there of Gaussian instead of MKS units and differing Fourier transform convention.

As was already noted elsewhere,[4] extra factors of (4π), $(2\pi)^{-6}$, and $(2\pi)^{-3}$ evidently enter through $E^{(2)}_{dpkn}$ in equation (19), due respectively to the use of Gaussian units, differing Fourier transform convention, and differing normalization of the background distribution $f^{R(0)}_{p_s}$ in equation (21). However the normalization in equation (24) of Akopyan and Tsytovich[1] is apparently the same as that here. Otherwise an additional factor of $(2\pi)^{-3}$ was omitted there in equation (24). A factor of $(2\pi)^3$ arises if the normalization is restored. An additional factor of $(4\pi)(2\pi)^3$ enters through $E^{(1)}_{kn}$ in equation (16) due to the use of Gaussian units and differing Fourier transform convention. As already noted, an additional factor of $(2\pi)^{-3}$ enters through equation (1) due to the differing Fourier transform convention. These factors combine to

$$\left[(4\pi)(2\pi)^{-6}(2\pi)^{-3}\right]\left[(2\pi)^3\right]\left[4\pi(2\pi)^3\right]\left[(2\pi)^{-3}\right] = (4\pi)^2(2\pi)^{-6} \quad .$$

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[1]*A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys. 1 (1975), 371].*

[4]*H. E. Brandt, The Total Field in Collective Bremsstrahlung In a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1996 (September 1983).*

Because the same definition of $E_k^{\sigma(0)}$ in equation (28) here is used in equation (13) there, the differing Fourier transform convention is compensated for. Thus equation (75) is in essential agreement with equation (24) of Akopyan and Tsytovich[1] up to the omission of a factor of $(-2e_\alpha)$ and a complex conjugation sign on the conductivity tensor. This follows since the δ in $\omega_1 + i\delta$ may clearly be ignored, and

$$\Sigma^{(s)}_{m\ell ij}\left(k_1,-k,k,k_1\right) = -iT^{(s)*}_{m\ell ij}\left(k_1,k,-k,-k_1\right) \quad . \tag{76}$$

Also if the background is assumed to be nondissipative to the needed order then it follows that

$$\left(\omega_1 + i\delta\right)^{-1}G_{nj}\left(k_1\right) = \left[\left(\omega_1 + i\delta\right)^{-1}G_{jn}\left(k_1\right)\right]^* \quad , \tag{77}$$

which is true under certain conditions to be discussed below. Also for a spatially isotropic system the Green's function is symmetric in its indices according to equation (7). It should, however, be noted by comparing equation (14) of Akopyan and Tsytovich[1] with equation (21) of Brandt[4] that the indices of G_{ij} are interchanged in Akopyan and Tsytovich[1] in the definition of the Green's function. It also follows that if one ignores the zero frequency resonance denominator, then equation (77) may be used provided the contribution of the imaginary part of the photonic Green's function is negligible to the needed order. Evidently the complex conjugation on the conductivity in equation (24) of Akopyan and Tsytovich[1] has been omitted. It is required if their equation (25) is to be consistent with the fact that the first and second complex denominators in their equation (21) are $\left(\omega - \left(\vec{k}\cdot\vec{v}_s\right) - i\delta\right)$ and $\left(\omega + \omega_1 - \left(\vec{k} + \vec{k}_1\right)\cdot\vec{v}_s - i\delta\right)$, respectively.[4] In equation (76) the tensor $T^{(s)}_{ij\ell m}\left(k,k_1,k_2,k_3\right)$ is given by[1,4]

$$\begin{aligned} T^{(s)}_{ij\ell m}\left(k,k_1,k_2,k_3\right) &= e_s^3 \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{v_{si}}{\omega - \vec{k}\cdot\vec{v}_s - i\delta} \left[\left(\omega_1 - \vec{k}_1\cdot\vec{v}_s\right) \frac{\partial}{\partial p_{sj}}\right. \\ &\left. + v_{sj}\left(\vec{k}_1\cdot\vec{\nabla}_{p_s}\right)\right] \left\{\frac{1}{\omega + \omega_1 - \left(\vec{k} + \vec{k}_1\right)\cdot\vec{v}_s - i\delta} \left[\left(\omega_2 - \vec{k}_2\cdot\vec{v}_s\right) \frac{\partial}{\partial p_{s\ell}}\right.\right. \\ &\left.\left. + v_{s\ell}\left(\vec{k}_2\cdot\vec{\nabla}_{p_s}\right)\right] \left[\frac{\partial}{\partial p_{sm}} + \frac{v_{sm}}{\omega_3 - \vec{k}_3\cdot\vec{v}_s + i\delta} \left(\vec{k}_3\cdot\vec{\nabla}_{p_s}\right)\right]\right\} f^{R(0)}_{p_s} \quad . \end{aligned} \tag{78}$$

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[1] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys. 1 (1975), 371].*

[4] *H. E. Brandt, The Total Field in Collective Bremsstrahlung In a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1996 (September 1983).*

It was concluded elsewhere[4] that for consistency, negative imaginary parts must be implicit in the first two denominators of equation (21) of Akopyan and Tsytovich.[1] By equation (22) here it is evident that

$$\Sigma_{ij\ell m}^{(s)}(k_1,-k,k,k_1) = -\Sigma_{ij\ell m}^{(s)*}(-k_1,k,-k,-k_1) \quad . \tag{79}$$

Also from equations (22) and (78) it follows that

$$\Sigma_{ij\ell m}^{(s)}(-k_1,k,-k,-k_1) = -iT_{ij\ell m}^{(s)}(k_1,k,-k,-k_1) \quad . \tag{80}$$

If we substitute equation (80) in equation (79), then equation (76) follows.

Equation (77) holds approximately if there is negligible linear absorption of energy by the background beam-plasma. To see this one notes that the energy $\varepsilon_{abs}^{(0)}$ absorbed linearly by the background beam-plasma is given by

$$\varepsilon_{abs}^{(0)} = \int d^3\vec{r} \int_{-\infty}^{\infty} dt\ \vec{E}(\vec{r},t)\cdot\vec{j}(\vec{r},t) \quad , \tag{81}$$

where the Fourier transform of the field is given by equation (21) in an earlier report,[4] namely

$$E_{ki} = \frac{-i}{\omega + i\delta} G_{ij}(k)j_{kj} \quad . \tag{82}$$

In equation (82) G_{ij} is the linear photon Green's function of the background beam-plasma. Rewriting equation (81) in terms of the Fourier transformed quantities one obtains

$$\varepsilon_{abs}^{(0)} = (2\pi)^4 \int dk\ E_{ki}\cdot j_{-ki} \quad . \tag{83}$$

Next substituting equation (82) in equation (83) produces

$$\varepsilon_{abs}^{(0)} = -i(2\pi)^4 \int \frac{dk}{\omega + i\delta} G_{ij}(k)j_{kj}j_{-ki} \quad . \tag{84}$$

If the current $\vec{j}$ is to be real, then

$$\vec{j}_{-k} = \vec{j}_k^* \quad . \tag{85}$$

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[4] *H. E. Brandt, The Total Field in Collective Bremsstrahlung In a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1996 (September 1983).*

Substituting equation (85) in equation (84) results in

$$\varepsilon_{abs}^{(0)} = -i(2\pi)^4 \int dk \left(\frac{G_{ij}(k)}{\omega + i\delta}\right) j_{kj} j_{ki}^* \quad . \tag{86}$$

Also, clearly, since the energy must be real, then

$$\varepsilon_{abs}^{(0)*} = \varepsilon_{abs}^{(0)} \quad . \tag{87}$$

Then using equation (87), we find

$$\varepsilon_{abs}^{(0)} = \frac{1}{2}\left(\varepsilon_{abs}^{(0)} + \varepsilon_{abs}^{(0)*}\right) \quad . \tag{88}$$

But using equation (86)

$$\varepsilon_{abs}^{(0)*} = i(2\pi)^4 \int \frac{dk}{\omega - i\delta} G_{ij}^*(k) j_{kj}^* j_{ki} \quad . \tag{89}$$

Or, if we rename dummy indices $\{i,j\}$ as $\{j,i\}$, equation (89) becomes

$$\varepsilon_{abs}^{(0)*} = -i(2\pi)^4 \int dk \left(\frac{-G_{ji}(k)}{\omega + i\delta}\right)^* j_{kj} j_{ki}^* \quad . \tag{90}$$

Then substituting equations (86) and (90) in equation (88), we get

$$\varepsilon_{abs}^{(0)} = -\frac{i}{2}(2\pi)^4 \int dk \left[\left(\frac{G_{ij}(k)}{\omega + i\delta}\right) - \left(\frac{G_{ji}(k)}{\omega + i\delta}\right)^*\right] j_{ki}^* j_{kj} \quad . \tag{91}$$

If linear absorption by the background is to be vanishing for arbitrary current $\vec{j}_k$, then equation (91) must be vanishing, and therefore

$$\frac{G_{ij}(k)}{\omega + i\delta} = \left(\frac{G_{ji}(k)}{\omega + i\delta}\right)^* , \tag{92}$$

in which case equation (77) immediately follows. It is to be stressed that equation (77) is at best an approximation since the background will not be completely linearly nondissipative. In summary then, equation (75) is in essential agreement with equation (24) of Akopyan and Tsytovich[1] except for a typographical error of omission of a factor of $-2e_\alpha$ in the latter; also Akopyan and Tsytovich there implicitly assume principal values with respect to single-wave particle resonance denominators as well as the approximate nondissipation relation, equation (77). However it is to be stressed that equation (75) is independent of these assumptions.

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[1] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys. 1 (1975), 371].*

Proceeding with the further reduction of equation (75), one first defines

$$\bar{\Sigma}_{m\ell ij}(k_1,-k,k,k_1) \equiv B \sum_s e_s \left[\Sigma^{(s)}_{m\ell ij}(k_1,-k,k,k_1) + \Sigma^{(s)}_{m\ell ji}(k_1,-k,k_1,k) \right] \quad . \tag{93}$$

Again the B operation selects only the part leading to the bremsstrahlung form given by equation (11). Using equation (93) in equation (75) produces

$$\begin{aligned}\langle \vec{F}_\alpha^{\sigma rad(1a)} \rangle = B \left(\frac{-e_\alpha^2}{16\pi^3} \right) \int \frac{dk\ dk_1\ \vec{k}_1}{\omega^2(\omega_1 + i\delta)^4} \left| E_k^{\sigma(0)} \right|^2 \\ \times v_{\alpha p} G_{pm}(k_1) v_{\alpha n} G_{jn}(k_1) e^{\sigma *}_{k\ell} e^{\sigma}_{ki} \delta(\omega_1 - \vec{k}_1 \cdot \vec{v}_\alpha) \\ \times \bar{\Sigma}_{m\ell ij}(k_1,-k,k,k_1) \quad .\end{aligned} \tag{94}$$

If we substitute equation (73) in equation (22), then equation (93) becomes

$$\begin{aligned}&\bar{\Sigma}_{m\ell ij}(k_1,-k,k,k_1) \\ &= B(-i) \sum_s e_s^4 \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{v_{sm}}{\omega_1 - \vec{k}_1 \cdot \vec{v}_s + i\delta} \\ &\times \left[(-\omega + \vec{k}\cdot\vec{v}_s)\nabla_{p_{s\ell}} + v_{s\ell}(-\vec{k}\cdot\vec{\nabla}_{p_s}) \right] \left[P \frac{1}{\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_s} \right. \\ &\left. - i\pi\delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_s\right) \right] \left\{ \left[(\omega - \vec{k}\cdot\vec{v}_s)\nabla_{p_{si}} + v_{si}(\vec{k}\cdot\vec{\nabla}_{p_s}) \right] \right. \\ &\times \left[\nabla_{p_{sj}} + \frac{v_{sj}}{\omega_1 - \vec{k}_1\cdot\vec{v}_s + i\delta} (\vec{k}_1\cdot\vec{\nabla}_{p_s}) \right] \\ &+ \left[(\omega_1 - \vec{k}_1\cdot\vec{v}_s)\nabla_{p_{sj}} + v_{sj}(\vec{k}_1\cdot\vec{\nabla}_{p_s}) \right] \\ &\left. \times \left[\nabla_{p_{si}} + \frac{v_{si}}{\omega - \vec{k}\cdot\vec{v}_s + i\delta} \vec{k}\cdot\vec{\nabla}_{p_s} \right] \right\} f_{p_s}^{R(0)} \quad .\end{aligned} \tag{95}$$

If we compare equations (94) and (95) with equation (11), it is evident that only the part of equation (95) proportional to $\delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right)$ can lead to the bremsstrahlung form given by equation (11), which is proportional to $\delta\left(\omega - \vec{k}\cdot\vec{v}_\alpha + (\vec{\kappa} - \vec{k})\cdot\vec{v}_\beta\right)$. Note that because of the factor $\delta\left(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha\right)$ in equation (94) one can set $\omega_1 = \vec{k}_1\cdot\vec{v}_\alpha$ in $\delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right)$ and also rename the integration variable $\vec{k}_1$ as $-\vec{\kappa}$ to obtain $\delta\left(\omega - \vec{k}\cdot\vec{v}_\alpha + (\vec{\kappa} - \vec{k})\cdot\vec{v}_\beta\right)$. Therefore the B operation in equation (95) selects, out of the sum over s, only the term $s = \beta$, and also only that part proportional to $\delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right)$. Therefore equation (95) becomes

$$\bar{\Sigma}_{m\ell ij} = B(-i)e_\beta^4 \int \frac{d^3\vec{p}_\beta}{(2\pi)^3} \frac{v_{\beta m}}{\omega_1 - \vec{k}_1\cdot\vec{v}_\beta + i\delta} \Big[\big(-\omega + \vec{k}\cdot\vec{v}_\beta\big)\nabla_{p_{\beta\ell}} \tag{96}$$

$$+ v_{\beta\ell}\big(-\vec{k}\cdot\vec{\nabla}_{p_\beta}\big)\Big](-i\pi)D_{ij}\big(k,k_1,\vec{p}_\beta\big)f_{p_\beta}^{R(0)} \quad ,$$

where the operator $D_{ij}\big(k,k_1,\vec{p}_\beta\big)$ is defined by

$$D_{ij}\big(k,k_1,\vec{p}_\beta\big)$$

$$\equiv \delta\big(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\big)\Bigg\{\Big[\big(\omega - \vec{k}\cdot\vec{v}_\beta\big)\nabla_{p_{\beta i}}$$

$$+ v_{\beta i}\big(\vec{k}\cdot\vec{\nabla}_{p_\beta}\big)\Big]\left[\nabla_{p_{\beta j}} + \frac{v_{\beta j}}{\omega_1 - \vec{k}_1\cdot\vec{v}_\beta + i\delta}\,\vec{k}_1\cdot\vec{\nabla}_{p_\beta}\right] \tag{97}$$

$$+ \Big[\big(\omega_1 - \vec{k}_1\cdot\vec{v}_\beta\big)\nabla_{p_{\beta j}} + v_{\beta j}\big(\vec{k}_1\cdot\vec{\nabla}_{p_\beta}\big)\Big]\left[\nabla_{p_{\beta i}} + \frac{v_{\beta i}}{\omega - \vec{k}\cdot\vec{v}_\beta + i\delta}\,\vec{k}\cdot\vec{\nabla}_{p_\beta}\right]\Bigg\} .$$

Simplifying equations (96) and (97) produces

$$\bar{\Sigma}_{m\ell ij} = B\pi e_\beta^4 \int \frac{d^3\vec{p}_\beta}{(2\pi)^3} \frac{v_{\beta m}}{\omega_1 - \vec{k}_1\cdot\vec{v}_\beta + i\delta} \Big[\big(\omega - \vec{k}\cdot\vec{v}_\beta\big)\nabla_{p_{\beta\ell}}$$

$$+ v_{\beta\ell}\vec{k}\cdot\vec{\nabla}_{p_\beta}\Big]D_{ij}f_{p_\beta}^{R(0)} \tag{98}$$

and

$$D_{ij} = \delta\big(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\big)\Bigg\{\big(\omega - \vec{k}\cdot\vec{v}_\beta\big)\frac{\partial^2}{\partial p_{\beta i}\partial p_{\beta j}} + \big(\omega_1 - \vec{k}_1\cdot\vec{v}_\beta\big)\frac{\partial^2}{\partial p_{\beta j}\partial p_{\beta i}}$$

$$+ \big(\omega - \vec{k}\cdot\vec{v}_\beta\big)\nabla_{p_{\beta i}}\frac{v_{\beta j}}{\omega_1 - \vec{k}_1\cdot\vec{v}_\beta + i\delta}\big(\vec{k}_1\cdot\vec{\nabla}_{p_\beta}\big) + v_{\beta i}\big(\vec{k}\cdot\vec{\nabla}_{p_\beta}\big)\nabla_{p_{\beta j}} \tag{99}$$

$$+ v_{\beta i}\big(\vec{k}\cdot\vec{\nabla}_{p_\beta}\big)\frac{v_{\beta j}}{\omega_1 - \vec{k}_1\cdot\vec{v}_\beta + i\delta}\big(\vec{k}_1\cdot\vec{\nabla}_{p_\beta}\big) + \big(\omega_1 - \vec{k}_1\cdot\vec{v}_\beta\big)\nabla_{p_{\beta j}}\frac{v_{\beta i}}{\omega - \vec{k}\cdot\vec{v}_\beta + i\delta}$$

$$\times \big(\vec{k}\cdot\vec{\nabla}_{p_\beta}\big) + v_{\beta j}\big(\vec{k}_1\cdot\vec{\nabla}_{p_\beta}\big)\nabla_{p_{\beta i}} + v_{\beta j}\big(\vec{k}_1\cdot\vec{\nabla}_{p_\beta}\big)\frac{v_{\beta i}}{\omega - \vec{k}\cdot\vec{v}_\beta + i\delta}\big(\vec{k}\cdot\vec{\nabla}_{p_\beta}\big)\Bigg\} \quad ,$$

where the arguments of $D_{ij}\big(k,k_1,\vec{p}_\beta\big)$ are suppressed. If we use the property of the delta function in equation (99) to replace $\omega_1 - \vec{k}_1\cdot\vec{v}_\beta$ by $-\omega + \vec{k}\cdot\vec{v}_\beta$, the first two terms cancel and equation (99) becomes

$$D_{ij} = \delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right)\left[(\omega - \vec{k}\cdot\vec{v}_\beta)\nabla_{p_{\beta i}} \frac{v_{\beta j}}{\omega_1 - \vec{k}_1\cdot \vec{v}_\beta + i\delta} \vec{k}_1\cdot\vec{\nabla}_{p_\beta}\right.$$
$$+ v_{\beta i}(\vec{k}\cdot\vec{\nabla}_{p_\beta})\nabla_{p_{\beta j}} + v_{\beta i}(\vec{k}\cdot\vec{\nabla}_{p_\beta}) \frac{v_{\beta j}}{\omega_1 - \vec{k}_1\cdot\vec{v}_\beta + i\delta} (\vec{k}_1\cdot\vec{\nabla}_{p_\beta}) \tag{100}$$
$$- (\omega - \vec{k}\cdot\vec{v}_\beta)\nabla_{p_{\beta j}} \frac{v_{\beta i}}{\omega - \vec{k}\cdot\vec{v}_\beta + i\delta} (\vec{k}\cdot\vec{\nabla}_{p_\beta})$$
$$\left. + v_{\beta j}(\vec{k}_1\cdot\vec{\nabla}_{p_\beta})\nabla_{p_{\beta i}} + v_{\beta j}(\vec{k}_1\cdot\vec{\nabla}_{p_\beta}) \frac{v_{\beta i}}{\omega - \vec{k}\cdot\vec{v}_\beta + i\delta} (\vec{k}\cdot\vec{\nabla}_{p_\beta})\right] .$$

Using the properties of the derivative operator and using the delta function, one may rewrite equation (100) as

$$D_{ij} = \delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right)\left\{(\omega - \vec{k}\cdot\vec{v}_\beta)\left[\nabla_{p_{\beta i}}\left(\frac{v_{\beta j}}{\omega_1 - \vec{k}_1\cdot\vec{v}_\beta + i\delta}\right)\right]\vec{k}_1\cdot\vec{\nabla}_{p_\beta}\right.$$
$$- v_{\beta j}(\vec{k}_1\cdot\vec{\nabla}_{p_\beta})\nabla_{p_{\beta i}} + v_{\beta i}(\vec{k}\cdot\vec{\nabla}_{p_\beta})\nabla_{p_{\beta j}} + v_{\beta i}(\vec{k}\cdot\vec{\nabla}_{p_\beta})\left(\frac{v_{\beta j}}{\omega_1 - \vec{k}_1\cdot\vec{v}_\beta + i\delta}\right) \vec{k}_1\cdot\vec{\nabla}_{p_\beta} \tag{101}$$
$$- (\omega - \vec{k}\cdot\vec{v}_\beta)\left[\nabla_{p_{\beta j}}\left(\frac{v_{\beta i}}{\omega - \vec{k}\cdot\vec{v}_\beta + i\delta}\right)\right]\vec{k}\cdot\vec{\nabla}_{p_\beta} - v_{\beta i}(\vec{k}\cdot\vec{\nabla}_{p_\beta})\nabla_{p_{\beta j}}$$
$$\left. + v_{\beta j}(\vec{k}_1\cdot\vec{\nabla}_{p_\beta})\nabla_{p_{\beta i}} + v_{\beta j}(\vec{k}_1\cdot\vec{\nabla}_{p_\beta}) \frac{v_{\beta i}}{\omega - \vec{k}\cdot\vec{v}_\beta + i\delta} (\vec{k}\cdot\vec{\nabla}_{p_\beta})\right\} .$$

In equation (101) the second and seventh terms cancel and the third and sixth terms cancel. Further simplifying equation (101) produces

$$D_{ij} = \delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right)\left\{(\omega - \vec{k}\cdot\vec{v}_\beta)\left[\nabla_{p_{\beta i}}\left(\frac{v_{\beta j}}{\omega_1 - \vec{k}_1\cdot\vec{v}_\beta + i\delta}\right)\right]\vec{k}_1\cdot\vec{\nabla}_{p_\beta}\right.$$
$$- (\omega - \vec{k}\cdot\vec{v}_\beta)\left[\nabla_{p_{\beta j}}\left(\frac{v_{\beta i}}{\omega - \vec{k}\cdot\vec{v}_\beta + i\delta}\right)\right]\vec{k}\cdot\vec{\nabla}_{p_\beta} \tag{102}$$
$$+ v_{\beta i}\ \vec{k}\cdot\vec{\nabla}_{p_\beta}\left(\frac{v_{\beta j}}{\omega_1 - \vec{k}_1\cdot\vec{v}_\beta + i\delta}\right)\Bigg]\vec{k}_1\cdot\vec{\nabla}_{p_\beta}$$
$$\left. + v_{\beta j}\left[\vec{k}_1\cdot\vec{\nabla}_{p_\beta}\left(\frac{v_{\beta i}}{\omega - \vec{k}\cdot\vec{v}_\beta + i\delta}\right)\right]\vec{k}\cdot\vec{\nabla}_{p_\beta} - 2\pi i\delta(\omega_1 - \vec{k}_1\cdot\vec{v}_\beta)v_{\beta i}v_{\beta j}\vec{k}\cdot\vec{\nabla}_{p_\beta}\vec{k}_1\cdot\vec{\nabla}_{p_\beta}\right\}$$

where equation (73) has been used in obtaining the last term. The last term may however be dropped since it does not lead to the bremsstrahlung form, equation (11), and therefore it will not contribute to equation (94). In particular, because of the factor $\delta(\omega_1 - \vec{k}_1\cdot\vec{v}_\beta)$, the needed factor $\delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right)$ becomes $\delta(\omega - \vec{k}\cdot\vec{v}_\beta)$. Also, if we replace $\omega - \vec{k}\cdot\vec{v}_\beta$ by $-\omega_1 + \vec{k}_1\cdot\vec{v}_\beta$ in the first and $\omega_1 - \vec{k}_1\cdot\vec{v}_\beta$ by $-\omega + \vec{k}\cdot\vec{v}_\beta$ in the third term, equation (102) can be rewritten as

$$D_{ij} = \delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right)\left\{\left(\left(-\nabla_{p_{\beta i}} v_{\beta j}\right.\right.\right.$$

$$- (\omega_1 - \vec{k}_1\cdot\vec{v}_\beta) v_{\beta j}\left[\nabla_{p_{\beta i}}\left(\frac{1}{\omega_1 - \vec{k}_1\cdot\vec{v}_\beta + i\delta}\right)\right]\Bigg)\vec{k}_1\cdot\vec{\nabla}_{p_\beta}$$

$$- (\omega - \vec{k}\cdot\vec{v}_\beta)\left[\nabla_{p_{\beta j}}\left(\frac{v_{\beta i}}{\omega - \vec{k}\cdot\vec{v}_\beta + i\delta}\right)\right]\vec{k}\cdot\vec{\nabla}_{p_\beta} \tag{103}$$

$$+ v_{\beta i}\left(\left(\frac{1}{-\omega + \vec{k}\cdot\vec{v}_\beta + i\delta}\right)(\vec{k}\cdot\vec{\nabla}_{p_\beta} v_{\beta j}) + v_{\beta j}\left[\vec{k}\cdot\vec{\nabla}_{p_\beta}\left(\frac{1}{\omega_1 - \vec{k}_1\cdot\vec{v}_\beta + i\delta}\right)\right]\right)\vec{k}_1\cdot\vec{\nabla}_{p_\beta}$$

$$\left. + v_{\beta j}\left[\vec{k}_1\cdot\vec{\nabla}_{p_\beta}\left(\frac{v_{\beta i}}{\omega - \vec{k}\cdot\vec{v}_\beta + i\delta}\right)\right]\vec{k}\cdot\vec{\nabla}_{p_\beta}\right\} .$$

Using the relativistic relation

$$v_{\beta i} = p_{\beta i} c^2 [p_\beta^2 c^2 + (m_\beta c^2)^2]^{-1/2} \tag{104}$$

produces

$$\nabla_{p_{\beta j}} v_{\beta i} = \frac{1}{\gamma_\beta m_\beta}\left(\delta_{ij} - \frac{v_{\beta i} v_{\beta j}}{c^2}\right) , \tag{105}$$

where

$$\gamma_\beta = \left[1 + \left(\frac{p_\beta}{m_\beta c}\right)^2\right]^{1/2} . \tag{106}$$

Performing the differentiations in equation (103) using equation (105) produces

$$D_{ij} = \delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right)\left\{\left[-\frac{1}{\gamma_\beta m_\beta}\left(\delta_{ij} - \frac{v_{\beta i} v_{\beta j}}{c^2}\right)\right.\right.$$

$$- \left(\frac{k_{1\ell} v_{\beta j}}{\omega_1 - \vec{k}_1\cdot\vec{v}_\beta + i\delta}\right)\frac{1}{\gamma_\beta m_\beta}\left(\delta_{\ell i} - \frac{v_{\beta i} v_{\beta \ell}}{c^2}\right) + \left(\frac{v_{\beta i} k_\ell}{-\omega + \vec{k}\cdot\vec{v}_\beta + i\delta}\right)\frac{1}{\gamma_\beta m_\beta} \tag{107}$$

$$\left. \times \left(\delta_{\ell j} - \frac{v_{\beta \ell} v_{\beta j}}{c^2}\right) + \frac{v_{\beta i} k_{1\ell} k_m v_{\beta j}\left(\delta_{\ell m} - \frac{v_{\beta \ell} v_{\beta m}}{c^2}\right)}{(\omega_1 - \vec{k}_1\cdot\vec{v}_\beta + i\delta)^2 \gamma_\beta m_\beta}\right]\vec{k}_1\cdot\vec{\nabla}_{p_\beta}$$

$$+ \left[-\frac{1}{\gamma_\beta m_\beta}\left(\delta_{ij} - \frac{v_{\beta i} v_{\beta j}}{c^2}\right) - \frac{v_{\beta i} k_\ell}{\omega - \vec{k}\cdot\vec{v}_\beta + i\delta}\frac{1}{\gamma_\beta m_\beta}\left(\delta_{\ell j} - \frac{v_{\beta \ell} v_{\beta j}}{c^2}\right)\right.$$

$$\left.\left. + \frac{v_{\beta j} k_{1\ell}}{\omega - \vec{k}\cdot\vec{v}_\beta + i\delta}\frac{1}{\gamma_\beta m_\beta}\left(\delta_{i\ell} - \frac{v_{\beta \ell} v_{\beta i}}{c^2}\right) + \frac{v_{\beta j} v_{\beta i} k_{1\ell} k_m\left(\delta_{m\ell} - \frac{v_{\beta m} v_{\beta \ell}}{c^2}\right)}{(\omega - \vec{k}\cdot\vec{v}_\beta + i\delta)^2 \gamma_\beta m_\beta}\right]\vec{k}\cdot\vec{\nabla}_{p_\beta}\right\} .$$

Using the delta function to replace $\omega_1 - \vec{k}_1 \cdot \vec{v}_\beta$ by $-\omega + \vec{k} \cdot \vec{v}_\beta$ in the denominator, we can rewrite equation (107) as

$$D_{ij} = \delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1) \cdot \vec{v}_\beta\right) \left\{ \frac{1}{\gamma_\beta m_\beta\left(-\omega + \vec{k} \cdot \vec{v}_\beta + i\delta\right)} \left[\left(\delta_{ij} - \frac{v_{\beta i} v_{\beta j}}{c^2}\right)(\omega - \vec{k} \cdot \vec{v}_\beta) \right.\right.$$

$$- v_{\beta j}\left(k_{1i} - \frac{\vec{k}_1 \cdot \vec{v}_\beta v_{\beta i}}{c^2}\right) + v_{\beta i}\left(k_j - \frac{\vec{k} \cdot \vec{v}_\beta}{c^2} v_{\beta j}\right)$$

$$\left. + \frac{v_{\beta j} v_{\beta i}}{-\omega + \vec{k} \cdot \vec{v}_\beta + i\delta}\left(\vec{k} \cdot \vec{k}_1 - \frac{(\vec{k} \cdot \vec{v}_\beta)(\vec{k}_1 \cdot \vec{v}_\beta)}{c^2}\right)\right] \vec{k}_1 \cdot \vec{\nabla}_{p_\beta} \tag{108}$$

$$- \frac{1}{\gamma_\beta m_\beta\left(\omega - \vec{k} \cdot \vec{v}_\beta + i\delta\right)} \left[\left(\delta_{ij} - \frac{v_{\beta i} v_{\beta j}}{c^2}\right)(\omega - \vec{k} \cdot \vec{v}_\beta) + v_{\beta i}\left(k_j - \frac{\vec{k} \cdot \vec{v}_\beta}{c^2} v_{\beta j}\right) \right.$$

$$\left.\left. - v_{\beta j}\left(k_{1i} - \frac{\vec{k}_1 \cdot \vec{v}_\beta v_{\beta i}}{c^2}\right) - \frac{v_{\beta j} v_{\beta i}}{\omega - \vec{k} \cdot \vec{v}_\beta + i\delta}\left(\vec{k} \cdot \vec{k}_1 - \frac{(\vec{k} \cdot \vec{v}_\beta)(\vec{k}_1 \cdot \vec{v}_\beta)}{c^2}\right)\right] \vec{k} \cdot \vec{\nabla}_{p_\beta} \right\} .$$

When terms are combined, equation (108) becomes

$$D_{ij} = \delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1) \cdot \vec{v}_\beta\right) \left\{ - \frac{1}{\gamma_\beta m_\beta}\left(\delta_{ij} - \frac{v_{\beta i} v_{\beta j}}{c^2}\right)(\vec{k}_1 + \vec{k}) \cdot \vec{\nabla}_{p_\beta} \right.$$

$$- \frac{v_{\beta j} k_{1i}}{\gamma_\beta m_\beta\left(-\omega + \vec{k} \cdot \vec{v}_\beta + i\delta\right)} \vec{k}_1 \cdot \vec{\nabla}_{p_\beta} - \frac{\vec{k} \cdot \vec{v}_\beta}{\gamma_\beta m_\beta c^2} v_{\beta i} v_{\beta j} \left(\frac{\vec{k}_1}{-\omega + \vec{k} \cdot \vec{v}_\beta + i\delta}\right.$$

$$\left. + \frac{\vec{k}}{-\omega + \vec{k} \cdot \vec{v}_\beta - i\delta}\right) \cdot \vec{\nabla}_{p_\beta} + \frac{v_{\beta i} k_j}{\gamma_\beta m_\beta}\left(\frac{\vec{k}_1}{-\omega + \vec{k} \cdot \vec{v}_\beta + i\delta} + \frac{\vec{k}}{-\omega + \vec{k} \cdot \vec{v}_\beta - i\delta}\right) \cdot \vec{\nabla}_{p_\beta} \tag{109}$$

$$+ \frac{v_{\beta i} v_{\beta j}}{\gamma_\beta m_\beta}\left(\vec{k}_1 \cdot \vec{k} - \frac{\vec{k} \cdot \vec{v}_\beta \vec{k}_1 \cdot \vec{v}_\beta}{c^2}\right)\left(\frac{\vec{k}_1}{(\omega - \vec{k} \cdot \vec{v}_\beta - i\delta)^2} + \frac{\vec{k}}{(\omega - \vec{k} \cdot \vec{v}_\beta + i\delta)^2}\right) \cdot \vec{\nabla}_{p_\beta}$$

$$+ \frac{v_{\beta j} k_{1i}}{\gamma_\beta m_\beta\left(\omega - \vec{k} \cdot \vec{v}_\beta + i\delta\right)} \vec{k} \cdot \vec{\nabla}_{p_\beta}$$

$$\left. - \frac{v_{\beta i} v_{\beta j} \vec{k}_1 \cdot \vec{v}_\beta}{\gamma_\beta m_\beta c^2}\left(\frac{\vec{k}_1}{\omega - \vec{k} \cdot \vec{v}_\beta - i\delta} + \frac{\vec{k}}{\omega - \vec{k} \cdot \vec{v}_\beta + i\delta}\right) \cdot \vec{\nabla}_{p_\beta} \right\} .$$

Further simplifying equation (109) produces

$$D_{ij} = \delta\left(\omega + \omega_1 - \left(\vec{k} + \vec{k}_1\right)\cdot\vec{v}_\beta\right) \frac{1}{\gamma_\beta m_\beta} \left\{-\left(\delta_{ij} - \frac{v_{\beta i} v_{\beta j}}{c^2}\right)\left(\omega - \vec{k}\cdot\vec{v}_\beta\right)^2 \right.$$
$$+ \left[v_{\beta j} k_{1i} - v_{\beta i} k_j + \frac{v_{\beta i} v_{\beta j}}{c^2}\, \vec{v}_\beta\cdot\left(\vec{k} - \vec{k}_1\right)\right]\left(\omega - \vec{k}\cdot\vec{v}_\beta\right) \tag{110}$$
$$\left. + v_{\beta i} v_{\beta j}\left(\vec{k}_1\cdot\vec{k} - \frac{\vec{k}\cdot\vec{v}_\beta \vec{k}_1\cdot\vec{v}_\beta}{c^2}\right)\right\}$$
$$\times \left[\frac{\vec{k}}{\left(\omega - \vec{k}\cdot\vec{v}_\beta + i\delta\right)^2} + \frac{\vec{k}_1}{\left(\omega - \vec{k}\cdot\vec{v}_\beta - i\delta\right)^2}\right]\cdot\vec{\nabla}_{p_\beta} \quad .$$

Equation (110) may be rewritten as

$$D_{ij} = -\delta\left(\omega + \omega_1 - \left(\vec{k} + \vec{k}_1\right)\cdot\vec{v}_\beta\right) \frac{1}{\gamma_\beta m_\beta} \left\{\left(\omega - \vec{k}\cdot\vec{v}_\beta\right)^2 \delta_{ij} \right.$$
$$+ \left(\omega - \vec{k}\cdot\vec{v}_\beta\right)\left(v_{\beta i} k_j - v_{\beta j} k_{1i}\right) \tag{111}$$
$$- \frac{v_{\beta i} v_{\beta j}}{c^2}\left[\left(\omega - \vec{k}\cdot\vec{v}_\beta\right)^2 + \vec{v}_\beta\cdot\left(\vec{k} - \vec{k}_1\right)\left(\omega - \vec{k}\cdot\vec{v}_\beta\right)\right.$$
$$\left.\left. + \vec{k}\cdot\vec{k}_1 c^2 - \vec{k}\cdot\vec{v}_\beta \vec{k}_1\cdot\vec{v}_\beta\right]\right\} \left[\frac{\vec{k}}{\left(\omega - \vec{k}\cdot\vec{v}_\beta + i\delta\right)^2} + \frac{\vec{k}_1}{\left(\omega - \vec{k}\cdot\vec{v}_\beta - i\delta\right)^2}\right]\cdot\vec{\nabla}_{p_\beta} \quad .$$

Further simplifying equation (111) produces

$$D_{ij} = -\delta\left(\omega + \omega_1 - \left(\vec{k} + \vec{k}_1\right)\cdot\vec{v}_\beta\right) \frac{1}{\gamma_\beta m_\beta} \left\{\left(\omega - \vec{k}\cdot\vec{v}_\beta\right)^2 \delta_{ij} \right.$$
$$+ \left(\omega - \vec{k}\cdot\vec{v}_\beta\right)\left(v_{\beta i} k_j - v_{\beta j} k_{1i}\right) - \frac{v_{\beta i} v_{\beta j}}{c^2}\left[\vec{k}\cdot\vec{k}_1 c^2\right. \tag{112}$$
$$\left.\left. + \omega\left(\omega - \vec{k}_1\cdot\vec{v}_\beta - \vec{k}\cdot\vec{v}_\beta\right)\right]\right\} \left[\frac{\vec{k}}{\left(\omega - \vec{k}\cdot\vec{v}_\beta + i\delta\right)^2} + \frac{\vec{k}_1}{\left(\omega - \vec{k}\cdot\vec{v}_\beta - i\delta\right)^2}\right]\cdot\vec{\nabla}_{p_\beta} \quad .$$

Using the delta function to replace $\left(\omega - \vec{k}_1\cdot\vec{v}_\beta - \vec{k}\cdot\vec{v}_\beta\right)$ by $\left(-\omega_1\right)$ in equation (112), we obtain

$$D_{ij} = -\delta\left(\omega + \omega_1 - \left(\vec{k} + \vec{k}_1\right)\cdot\vec{v}_\beta\right) \frac{1}{\gamma_\beta m_\beta} \left[\left(\omega - \vec{k}\cdot\vec{v}_\beta\right)^2 \delta_{ij} \right.$$
$$\left. + \left(\omega - \vec{k}\cdot\vec{v}_\beta\right)\left(v_{\beta i} k_j - v_{\beta j} k_{1i}\right) - v_{\beta i} v_{\beta j}\left(\vec{k}\cdot\vec{k}_1 - \frac{\omega\omega_1}{c^2}\right)\right] \tag{113}$$
$$\times \left[\frac{\vec{k}}{\left(\omega - \vec{k}\cdot\vec{v}_\beta + i\delta\right)^2} + \frac{\vec{k}_1}{\left(\omega - \vec{k}\cdot\vec{v}_\beta - i\delta\right)^2}\right]\cdot\vec{\nabla}_{p_\beta} \quad .$$

Substituting equation (4) in equation (113) produces

$$D_{ij} = -\frac{1}{e_\beta}\,\delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right)$$
$$\times\left[\Lambda_{ij}^{(\beta)*}(k_1,k)\vec{k} + \Lambda_{ij}^{(\beta)}(k_1,k)\vec{k}_1\right]\cdot\vec{\nabla}_{p_\beta} \;. \tag{114}$$

Next substituting equation (114) in equation (98) results in

$$\bar{\Sigma}_{m\ell ij} = -B\pi e_\beta^3 \int \frac{d^3\vec{p}_\beta}{(2\pi)^3}\,\frac{v_{\beta m}}{\omega_1 - \vec{k}_1\cdot\vec{v}_\beta + i\delta}\left[(\omega - \vec{k}\cdot\vec{v}_\beta)\nabla_{p_{\beta\ell}}\right.$$
$$\left. + v_{\beta\ell}\vec{k}\cdot\vec{\nabla}_{p_\beta}\right]\delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right) \tag{115}$$
$$\times\left[\Lambda_{ij}^{(\beta)*}(k_1,k)\vec{k} + \Lambda_{ij}^{(\beta)}(k_1,k)\vec{k}_1\right]\cdot\vec{\nabla}_{p_\beta} f_{p_\beta}^{R(0)} \;.$$

Integrating equation (115) by parts, then (since the background distribution $f_{p_\beta}^{R(0)}$ is vanishing at infinity), one obtains

$$\bar{\Sigma}_{m\ell ij} = B\pi e_\beta^3 \int \frac{d^3\vec{p}_\beta}{(2\pi)^3}\left\{-\nabla_{p_{\beta\ell}}\left[\frac{(\omega - \vec{k}\cdot\vec{v}_\beta)v_{\beta m}}{-\omega_1 + \vec{k}_1\cdot\vec{v}_\beta - i\delta}\right]\right.$$
$$\left. - \nabla_{p_{\beta n}}\left[\frac{v_{\beta m}v_{\beta\ell}k_n}{-\omega_1 + \vec{k}_1\cdot\vec{v}_\beta - i\delta}\right]\right\}\delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right) \tag{116}$$
$$\times\left[\Lambda_{ij}^{(\beta)*}(k_1,k)\vec{k} + \Lambda_{ij}^{(\beta)}(k_1,k)\vec{k}_1\right]\cdot\vec{\nabla}_{p_\beta} f_{p_\beta}^{R(0)} \;.$$

Performing the derivatives in equation (116) using equation (105), and then using the delta function to replace $\omega_1 - \vec{k}_1\cdot\vec{v}_\beta$ by $-\omega + \vec{k}\cdot\vec{v}_\beta$ in the denominators, we obtain

$$\bar{\Sigma}_{m\ell ij} = B\pi e_\beta^3 \int \frac{d^3\vec{p}_\beta}{(2\pi)^3}\left\{-\frac{1}{\gamma_\beta m_\beta}\left(\delta_{m\ell} - \frac{v_{\beta m}v_{\beta\ell}}{c^2}\right) + \frac{k_n}{\omega - \vec{k}\cdot\vec{v}_\beta - i\delta}\,\frac{1}{\gamma_\beta m_\beta}\right.$$
$$\times\left(\delta_{n\ell} - \frac{v_{\beta n}v_{\beta\ell}}{c^2}\right)v_{\beta m} + \frac{1}{\omega - \vec{k}\cdot\vec{v}_\beta - i\delta}\,k_{1n}\,\frac{1}{\gamma_\beta m_\beta}\left(\delta_{n\ell} - \frac{v_{\beta n}v_{\beta\ell}}{c^2}\right)v_{\beta m}$$
$$- \frac{1}{\omega - \vec{k}\cdot\vec{v}_\beta - i\delta}\,\frac{1}{\gamma_\beta m_\beta}\left(\delta_{mn} - \frac{v_{\beta m}v_{\beta n}}{c^2}\right)v_{\beta\ell}k_n - \frac{v_{\beta m}}{\omega - \vec{k}\cdot\vec{v}_\beta - i\delta} \tag{117}$$
$$\left.\times\frac{1}{\gamma_\beta m_\beta}\left(\delta_{n\ell} - \frac{v_{\beta n}v_{\beta\ell}}{c^2}\right)k_n + \frac{v_{\beta m}v_{\beta\ell}k_n k_{1r}}{(\omega - \vec{k}\cdot\vec{v}_\beta - i\delta)^2}\,\frac{1}{\gamma_\beta m_\beta}\left(\delta_{nr} - \frac{v_{\beta n}v_{\beta r}}{c^2}\right)\right\}$$
$$\times\,\delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right)\left[\Lambda_{ij}^{(\beta)*}(k_1,k)\vec{k} + \Lambda_{ij}^{(\beta)}(k_1,k)\vec{k}_1\right]\cdot\vec{\nabla}_{p_\beta} f_{p_\beta}^{R(0)} \;.$$

Combining terms in equation (117) produces

$$\bar{\Sigma}_{m\ell ij} = B\pi e_\beta^3 \int \frac{d^3\vec{p}_\beta}{(2\pi)^3} \frac{1}{\gamma_\beta m_\beta} \Bigg[-\delta_{m\ell} - \frac{k_m v_{\beta\ell} - k_{1\ell} v_{\beta m}}{\omega - \vec{k}\cdot\vec{v}_\beta - i\delta} \tag{118}$$

$$+ \frac{v_{\beta m} v_{\beta\ell}}{c^2}\left(1 + \frac{(\vec{k} - \vec{k}_1)\cdot\vec{v}_\beta}{\omega - \vec{k}\cdot\vec{v}_\beta - i\delta} + \frac{\vec{k}\cdot\vec{k}_1 c^2}{(\omega - \vec{k}\cdot\vec{v}_\beta - i\delta)^2} - \frac{\vec{k}\cdot\vec{v}_\beta \vec{k}_1\cdot\vec{v}_\beta}{(\omega - \vec{k}\cdot\vec{v}_\beta - i\delta)^2}\right)\Bigg]$$

$$\times\ \delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right)\left[\Lambda_{ij}^{(\beta)*}(k_1,k)\vec{k} + \Lambda_{ij}^{(\beta)}(k_1,k)\vec{k}_1\right]\cdot\vec{\nabla}_{p_\beta} f_{p_\beta}^{R(0)} \quad .$$

Further simplifying equation (118), we obtain

$$\bar{\Sigma}_{m\ell ij} = B\pi e_\beta^3 \int \frac{d^3\vec{p}_\beta}{(2\pi)^3} \frac{1}{\gamma_\beta m_\beta} \Bigg[-\delta_{m\ell} - \frac{k_m v_{\beta\ell} - k_{1\ell} v_{\beta m}}{\omega - \vec{k}\cdot\vec{v}_\beta - i\delta}$$

$$+ \frac{v_{\beta m} v_{\beta\ell}}{c^2(\omega - \vec{k}\cdot\vec{v}_\beta - i\delta)^2}\left[\omega\left(\omega - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right) + \vec{k}\cdot\vec{k}_1 c^2\right]\Bigg]$$

$$\times\ \delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right)\left[\Lambda_{ij}^{(\beta)*}(k_1,k)\vec{k} \tag{119}\right.$$

$$\left. + \Lambda_{ij}^{(\beta)}(k_1,k)\vec{k}_1\right]\cdot\vec{\nabla}_{p_\beta} f_{p_\beta}^{R(0)} \quad .$$

If we use the delta function in equation (119) to replace $\left(\omega - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right)$ by $(-\omega_1)$, then after rearranging terms, we obtain

$$\bar{\Sigma}_{m\ell ij} = -B\pi e_\beta^3 \int \frac{d^3\vec{p}_\beta}{(2\pi)^3} \frac{1}{\gamma_\beta m_\beta} \Bigg[\delta_{\ell m} + \frac{v_{\beta\ell} k_m - v_{\beta m} k_{1\ell}}{\omega - \vec{k}\cdot\vec{v}_\beta - i\delta}$$

$$- \frac{v_{\beta\ell} v_{\beta m}}{(\omega - \vec{k}\cdot\vec{v}_\beta - i\delta)^2}\left(\vec{k}\cdot\vec{k}_1 - \frac{\omega\omega_1}{c^2}\right)\Bigg]\delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right) \tag{120}$$

$$\times\ \left[\Lambda_{ij}^{(\beta)*}(k_1,k)\vec{k} + \Lambda_{ij}^{(\beta)}(k_1,k)\vec{k}_1\right]\cdot\vec{\nabla}_{p_\beta} f_{p_\beta}^{R(0)} \quad .$$

Next substituting equation (4) in equation (120) produces

$$\bar{\Sigma}_{m\ell ij} = -B\pi e_\beta^2 \int \frac{d^3\vec{p}_\beta}{(2\pi)^3} \Lambda_{\ell m}^{(\beta)}(k_1,k)\delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right) \tag{121}$$

$$\times\ \left[\Lambda_{ij}^{(\beta)*}(k_1,k)\vec{k} + \Lambda_{ij}^{(\beta)}(k_1,k)\vec{k}_1\right]\cdot\vec{\nabla}_{p_\beta} f_{p_\beta}^{R(0)} \quad .$$

Equation (121) is to be compared with equation (26) of Akopyan and Tsytovich.[1] Since the B operation projects out s = β as discussed above, equation (93) may be written

$$\bar{\Sigma}_{m\ell ij}(k_1,-k,k,k_1) = Be_\beta[\Sigma^{(\beta)}_{m\ell ij}(k_1,-k,k,k_1) + \Sigma^{(\beta)}_{m\ell ji}(k_1,-k,k_1,k)] \quad . \tag{122}$$

Comparing equations (121) and (122), we find

$$\begin{aligned} &B[\Sigma^{(\beta)}_{m\ell ij}(k_1,-k,k,k_1) + \Sigma^{(\beta)}_{m\ell ji}(k_1,-k,k_1,k)] \\ &= -B\pi e_\beta \int \frac{d^3\vec{p}_\beta}{(2\pi)^3}\,\delta(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta) \\ &\quad \times \Lambda^{(\beta)}_{\ell m}(k_1,k)[\Lambda^{(\beta)*}_{ij}(k_1,k)\vec{k} + \Lambda^{(\beta)}_{ij}(k_1,k)\vec{k}_1]\cdot\vec{\nabla}_{p_\beta}f^{R(0)}_{p_\beta} \quad . \end{aligned} \tag{123}$$

By equation (4) it follows that[2]

$$\delta(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta)\Lambda^{(\beta)}_{\ell m}(k_1,k) = \delta(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta)\Lambda^{(\beta)*}_{m\ell}(k,k_1) \quad . \tag{124}$$

Substituting equations (76) and (124) in equation (123), dividing by i, and taking the complex conjugate, one obtains

$$\begin{aligned} &B[T^{(\beta)}_{m\ell ij}(k_1,k,-k,-k_1) + T^{(\beta)}_{m\ell ji}(k_1,k,-k_1,-k)] \\ &= Bi\pi e_\beta \int \frac{d^3p_\beta}{(2\pi)^3}\,\delta(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta)\Lambda^{(\beta)}_{m\ell}(k,k_1)[\Lambda^{(\beta)}_{ij}(k_1,k)\vec{k} \\ &\quad + \Lambda^{(\beta)*}_{ij}(k_1,k)\vec{k}_1]\cdot\vec{\nabla}_{p_\beta}f^{R(0)}_{p_\beta} \quad . \end{aligned} \tag{125}$$

If one ignores single-wave particle resonances, the iδ in equation (4) may be ignored and $\Lambda^{(\beta)}_{ij}(k_1,k)$ becomes effectively real and

$$P_1\Lambda^{(\beta)*}_{ij}(k_1,k) = P_1\Lambda^{(\beta)}_{ij}(k_1,k) \quad , \tag{126}$$

where P_1 denotes the operation of taking principal values with respect to Cerenkov resonance denominators of the form $(\omega - \vec{k}\cdot\vec{v}_\beta + i\delta)$. The principal part of $\Lambda^{(\beta)}_{ij}(k_1,k)$ is real. The nonprincipal part is proportional to $\delta(\omega - \vec{k}\cdot\vec{v}_\beta)$ or $\delta'(\omega - \vec{k}\cdot\vec{v}_\beta)$ according to equations (4) and (73). Therefore it will not yield the correct bremsstrahlung form and may be dropped. Operating on both sides of equation (125) with P_1 and using equation (126), we obtain

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[1] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys. 1 (1975), 371].*

[2] *H. E. Brandt, Nonlinear Force on an Unpolarized Relativistic Test Particle to Second-Order in the Total Force in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1995 (August 1983).*

$$P_1B\left[T^{(\beta)}_{m\ell ij}(k_1,k,-k,-k_1) + T^{(\beta)}_{m\ell ji}(k_1,k,-k_1,-k)\right]$$

$$= P_1Bi\pi e_\beta \int \frac{d^3\vec{p}_\beta}{(2\pi)^3}\, \delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right) \tag{127}$$

$$\times\, \Lambda^{(\beta)}_{m\ell}(k,k_1)\Lambda^{(\beta)}_{ij}(k_1,k)(\vec{k} + \vec{k}_1)\cdot\vec{\nabla}_{p_\beta} f^{R(0)}_{p_\beta} \quad .$$

Equation (127) agrees with equation (26) of Akopyan and Tsytovich[1] except for k appearing interchanged with k_1 in $\Lambda^{(\beta)}_{m\ell}$ and $\Lambda^{(\beta)}_{ij}$. This is apparently an error in the cited work.[1] The operation P_1B is implicit there and the i is absent since the imaginary part has been taken (the coefficient of i) and the right-hand side of equation (127) here is purely imaginary.

Next substituting equation (121) in equation (94) results in

$$\langle \vec{F}^{\sigma rad(1a)}_\alpha \rangle = B\,\frac{e^2_\alpha e^2_\beta}{16\pi^2}\int \frac{d^3\vec{p}_\beta}{(2\pi)^3}\,\frac{dk\ dk_1}{\omega^2(\omega_1 + i\delta)^4}$$

$$\times\, \vec{k}_1\left|E^{\sigma(0)}_k\right|^2\delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right)\delta\left(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha\right)$$

$$\times\, v_{\alpha p}G_{pm}(k_1)\Lambda^{(\beta)}_{\ell m}(k_1,k)e^{\sigma *}_{k\ell}\left[e^{\sigma}_{ki}\Lambda^{(\beta)*}_{ij}(k_1,k)G_{jn}(k_1)v_{\alpha n}\vec{k}\right. \tag{128}$$

$$\left. +\, e^{\sigma}_{ki}\Lambda^{(\beta)}_{ij}(k_1,k)G_{jn}(k_1)v_{\alpha n}\vec{k}_1\right]\cdot\vec{\nabla}_{p_\beta} f^{R(0)}_{p_\beta} \quad .$$

Using equations (92) and (124) in equation (128) produces

$$\langle \vec{F}^{\sigma rad(1a)}_\alpha \rangle$$

$$= B\,\frac{e^2_\alpha e^2_\beta}{16\pi^2}\int \frac{d^3\vec{p}_\beta}{(2\pi)^3}\,\frac{dk\ dk_1}{\omega^2(\omega_1 + i\delta)^2\omega_1^2}\,\vec{k}_1\left|E^{\sigma(0)}_k\right|^2$$

$$\times\, \delta\left(\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_\beta\right)\delta\left(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha\right)\left[e^{\sigma}_{k\ell}\Lambda^{(\beta)}_{m\ell}(k,k_1)G_{mp}(k_1)v_{\alpha p}\right]^* \tag{129}$$

$$\times\, \left[e^{\sigma}_{ki}\Lambda^{(\beta)}_{ji}(k,k_1)G_{jn}(k_1)v_{\alpha n}\vec{k} + e^{\sigma}_{ki}\Lambda^{(\beta)*}_{ji}(k,k_1)G_{jn}(k_1)v_{\alpha n}\vec{k}_1\right]\cdot\vec{\nabla}_{p_\beta} f^{R(0)}_{p_\beta} \quad .$$

Integrating over ω_1, letting $\vec{k}_1 = -\vec{k}$, and denoting

$$\kappa = \left(\vec{k}\cdot\vec{v}_\alpha, \vec{k}\right) \quad , \tag{130}$$

then we find that equation (129) becomes

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[1] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys. 1 (1975), 371].*

$$\langle \vec{F}_\alpha^{\sigma rad(1a)} \rangle = -B \frac{e_\alpha^2 e_\beta^2}{16\pi^2} \int \frac{d^3\vec{p}_\beta}{(2\pi)^3} \frac{d\omega\, d^3\vec{k}\, d^3\vec{\kappa}\, \vec{\kappa} |E_k^{\sigma(0)}|^2}{\omega^2(\vec{\kappa}\cdot\vec{v}_\alpha - i\delta)^2(\vec{\kappa}\cdot\vec{v}_\alpha)^2}$$

$$\times\ \delta\left(\omega - \vec{\kappa}\cdot\vec{v}_\alpha + (\vec{\kappa} - \vec{k})\cdot\vec{v}_\beta\right)\left[e_{k\ell}^{\sigma}\Lambda_{m\ell}^{(\beta)}(k,-\kappa)G_{mp}(-\kappa)v_{\alpha p}\right]^* \tag{131}$$

$$\times\ \left[e_{ki}^{\sigma}\Lambda_{ji}^{(\beta)}(k,-\kappa)G_{jn}(-\kappa)v_{\alpha n}\vec{k}\right.$$

$$\left. - e_{ki}^{\sigma}\Lambda_{ji}^{(\beta)*}(k,-\kappa)G_{jn}(-\kappa)v_{\alpha n}\vec{\kappa}\right]\cdot\vec{\nabla}_{p_\beta} f_{p_\beta}^{R(0)} \quad .$$

If Cerenkov resonance is ignored, equation (126) applies. This is justified since only the principal part of $\Lambda_{ij}^{(\beta)}(k,-\kappa)$ leads to the bremsstrahlung form, and the principal part is real. Also, ignoring the $i\delta$ in $\left(\vec{\kappa}\cdot\vec{v}_\alpha - i\delta\right)^{-1}$, dropping the unneeded B operation, and making the change of indices from $\{i,j,n,p\}$ to $\{j,i,p,n\}$, we find that equation (131) becomes

$$\langle \vec{F}_\alpha^{\sigma rad(1a)} \rangle = \frac{e_\alpha^2 e_\beta^2}{16\pi^2} \int \frac{d^3\vec{p}_\beta}{(2\pi)^3} \frac{d\omega\, d^3\vec{k}\, d^3\vec{\kappa}\, \vec{\kappa} |E_k^{\sigma(0)}|^2}{\omega^2(\vec{\kappa}\cdot\vec{v}_\alpha)^4}$$

$$\times\ \delta\left(\omega - \vec{\kappa}\cdot\vec{v}_\alpha + (\vec{\kappa} - \vec{k})\cdot\vec{v}_\beta\right) \tag{132}$$

$$\times\ \left|e_{kj}^{\sigma}\Lambda_{ij}^{(\beta)}(k,-\kappa)G_{ip}(-\kappa)v_{\alpha p}\right|^2(\vec{\kappa} - \vec{k})\cdot\vec{\nabla}_{p_\beta} f_{p_\beta}^{R(0)} \quad .$$

Equation (132) is in fact equation (3), as was to be shown. It is to be stressed that to obtain this expression it was necessary to assume that the background is approximately nonabsorptive and spatially isotropic. However, these assumptions are not involved in equation (128). As already stated in the introduction, equation (27) of Akopyan and Tsytovich[1] is apparently in error; namely, a factor of 2 is omitted there and the arguments of $\Lambda_{ij}^{(\beta)}(-\kappa,k)$ are interchanged. The additional factors of $(4\pi)^2(2\pi)^{-6}$ there are apparently due to the use of Gaussian units and different Fourier transform conventions, as already discussed in equation (75). Also the Green's function used there is symmetric.

3. CONCLUSION

A number of important physical assumptions and mathematical techniques have been identified which are involved in Tsytovich's theory of nonlinear bremsstrahlung and radiative instability in relativistic nonequilibrium beam-plasma systems. In particular, the stochastic average of the lowest-order Born approximation for the nonlinear force on the bare charge of a relativistic test particle, equation (1), has been reduced to the form given by equations (2), (3), (47), (25), and (26). In the result of Akopyan and Tsytovich,[1] their equation (27) which corresponds to equation (3) here, a factor of 2 is evidently omitted and the arguments of $\Lambda_{ij}^{(\beta)}$ are incorrectly interchanged. Equation (3) has been shown to hold provided the background is approximately nonabsorptive and spatially isotropic. However, the form given by equation (128) holds without these assumptions. Also, an important expression, equation (123), was obtained involving the third-order nonlinear conductivity tensor. Using this expression it was shown that in equation (26) of Akopyan and Tsytovich[1] the arguments of $\Lambda_{ij}^{(\beta)}$ are again incorrectly interchanged.

The present work will facilitate better understanding of the methods necessary to reduce the other components of the total nonlinear force due to bremsstrahlung emission.[1] The latter is needed to determine the nonlinear bremsstrahlung probability and the conditions for the occurrence of a bremsstrahlung radiative instability in nonequilibrium relativistic beam-plasma systems.

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[1] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys. 1 (1975), 371].*

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Other related work prepared in preprint form will be published later and is available from the author.

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